No. K-72/67

Technical Memorandum

BI-CHI: A COMPUTER PROGRAM FOR THE CHI-SQUARE GOODNESS OF FIT TEST FOR A BIVARIATE NORMAL DISTRIBUTION

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U. S. NAVAL WEAPONS LABORATORY

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While the contents of this memorandum are considered to be correct, they are subject to modification upon further study.

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ABSTRACT

Frequently the applied statistician is confronted with the problem of investigating the distribution of experimental data sampled from a continuous two-dimensional distribution. A prime example in the area of weapons system analysis is the investigation of the distribution of hit points of missiles or projectiles. Because of the desirable characteristics of the bivariate normal distribution and because of the frequency of occurrence of these characteristics in physical phenomena, the applied statistician is often interested in determining if the sample distribution of the experimental data can be approximated by a bivariate normal distribution. The BI-CHI (Bivariate Chi-Square Test) computer program described in this report is for assessing the agreement of a continuous two-dimensional sample distribution with a parent bivariate normal distribution. The program computes the Chi-square statistic for testing the null hypothesis "The random sample is from a bivariate normal parent population." The minimum and maximum sample values of each random variable are determined, and the sample estimates of the five parameters (two means, two variances, and the correlation coefficient) of the bivariate normal distribution are computed. Also, the sample estimates of the two regression lines (x_2 on x_1 and x_1 on x_2) are computed. An optional feature allows additional output of a plot of the sample data and the contour ellipses.

The program is coded in FORTRAN IV for the IBM 7030 (STRETCH) computer.

FOREWORD

The formulation of the BI-CHI computer program was performed in the Mathematical Statistics Branch of the Operations Research Division. The coding of the program was performed in the Operations Sciences Branch of the Computer Programming Division. The project was supported under Foundational Research Project No. 29Y, "Computer Programs for Statistical Analyses."

The original program, used for exploratory computations, was coded by Mr. Rick Ball; the final program as documented in this report was coded by Mrs. Jo Ann Brown. Miss Nancy Dean coded the computer routine which generated the c_p^2-values tabulated within the BI-CHI program.

The date of completion was 31 October 1967.

I. INTRODUCTION

BI-CHI (Bivariate Chi-Square Test) is a computer program for the Chi-Square Goodness of Fit Test for a bivariate normal distribution. The program performs a statistical analysis of the discrepancy between sampled data from a continuous two-dimensional distribution and the hypothesized bivariate normal distribution. This discrepancy is measured by classifying the n randomly sampled (x_{1i},x_{2i})-observations into k mutually exclusive and exhaustive categories (areas between coaxial contour ellipses) and comparing the observed frequency with the theoretical frequency of each of the k areas. The formulation of the Goodness of Fit Test is described in Bates (1966) and therefore is not repeated in its entirety in this report.

The joint probability density function (pdf) of two jointly normally distributed random variables x_1 and x_2 is

$$f(x_1,x_2) = K^{-1}e^{-\frac{1}{2}Q}; -\infty < x_1 < \infty, \ -\infty < x_2 < \infty, \tag{1}$$

where

$$Q = \frac{1}{(1-\rho^2)}\left[\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{x_1-\mu_1}{\sigma_1}\right)\left(\frac{x_2-\mu_2}{\sigma_2}\right) + \left(\frac{x_2-\mu_2}{\sigma_2}\right)^2\right]. \tag{2}$$

A horizontal plane parallel to the x_1x_2-plane intersects the surface of equation (1) in the shape of an ellipse (contour ellipse). The equation of the general contour ellipse is

$$Q = c^2, \tag{3}$$

where c is a constant. The contour ellipses are centered at (μ_1,μ_2), and their general shape is determined by σ_1,σ_2, and ρ. By performing an orthogonal transformation, the quadratic form (Q) can be shown to be distributed as χ^2 with two degrees of freedom. That is,

$$Q \sim \chi^2(2). \tag{4}$$

Therefore, the probability that (x_1,x_2) falls in the contour ellipse of equation (3) is

$$P[\chi^2(2) < c_p^2] = p, \tag{5}$$

where c_p^2 is defined by $p = \int_0^{c_p^2} f(\chi^2)d\chi^2$, where $f(\chi^2)$ is the pdf of $\chi^2(2)$. Additionally, the probability that (x_1,x_2) falls inside the p'' contour ellipse and outside the p' contour ellipse is

$$P[c_{p'}^2 \le \chi^2(2) < c_{p''}^2] = p''-p'; \; 0 \le p' < p'' \le 1. \tag{6}$$

The classification of the (x_{1i},x_{2i})-observations is then performed by evaluating $Q_i (i=1,2,\cdots,n)$. If $c_{p_{j-1}}^2 \le Q_i < c_{p_j}^2$ $(j=1,2,\cdots,k)$, the i^{th} observation falls within the j^{th} contour ellipse and outside the $(j-1)^{th}$ contour ellipse. Denoting the observed number of (x_{1i},x_{2i}) falling within the j^{th} contour ellipse and outside the $(j-1)^{th}$ contour ellipse by O_j and the expected number by $E_j = n(p_j-p_{j-1})$, the statistic

$$\sum_{j=1}^{k} (O_j-E_j)^2/E_j \tag{7}$$

is approximately distributed as $\chi^2(k-1)$.

In practice, however, the population parameters of the hypothesized bivariate normal distribution are unknown. That is, the population is not completely specified under the null hypothesis; only the type of the parent population is known or assumed. In such case, the population parameters are estimated from the sample data; Q_i is replaced with $\hat{Q}_i$; and equation (7) becomes

$$X^2 = \sum_{j=1}^{k} (0_j - \hat{E}_j)^2 / \hat{E}_j, \tag{8}$$

where $\hat{E}_j$ denotes the use of sample statistics to estimate population parameters. The degrees of freedom of X^2 is reduced by one for each parameter estimated. Therefore, when all five parameters of the bivariate normal distribution are estimated, X^2 is approximately distributed as $\chi^2(k-6)$.

BI-CHI computes the five estimators of the population parameters, classifies the $\hat{Q}_i (i=1,2,\cdots,n)$, and computes X^2. Estimators of the two "true" regression lines are computed, and the two minimum and maximum sample values are determined. In addition to the printout of the above computed statistics, optional features provide for printout of the categorization, a plot of the data, and a plot of the data with contour ellipses.

II. COMPUTATIONAL PROCEDURE

The estimators of the five population parameters are

$$\hat{\mu}_v = \bar{x}_v = \Sigma x_{vi}/n, \qquad \hat{\sigma}_v^2 = s_v^2 = \Sigma(x_{vi}-\bar{x}_v)^2/(n-1), \tag{9}$$

$$\hat{\rho} = r = \frac{\Sigma(x_{1i}-\bar{x}_1)(x_{2i}-\bar{x}_2)}{\sqrt{\Sigma(x_{1i}-\bar{x}_1)^2\ \Sigma(x_{2i}-\bar{x}_2)^2}},$$

where $v=1,2$ and all summation signs run from 1 to n. The minimum and maximum sample values of each random variable are the respective order statistics $x_{1(1)}$, $x_{1(n)}$ and $x_{2(1)}$, $x_{2(n)}$. Estimates of the two "true" regression lines are

$$\begin{aligned} x_2 &= \bar{x}_2 + r\,\frac{s_2}{s_1}(x_1-\bar{x}_1) = a_1 + b_1 x_1, \\ x_1 &= \bar{x}_1 + r\,\frac{s_1}{s_2}(x_2-\bar{x}_2) = a_2 + b_2 x_2. \end{aligned} \tag{10}$$

The five statistics of equation (9) are substituted for the five parameters of equation (2), giving

$$\hat{Q}_i \quad \frac{1}{(1-r^2)}\left[\left(\frac{x_{1i}-\bar{x}_1}{s_1}\right)^2 - 2r\left(\frac{x_{1i}-\bar{x}_1}{s_1}\right)\left(\frac{x_{2i}-\bar{x}_2}{s_2}\right) + \left(\frac{x_{2i}-\bar{x}_2}{s_2}\right)^2\right]. \tag{11}$$

After evaluating $\hat{Q}_i$ $(i=1,2,\cdots,n)$, the classification of the n observations is performed by comparing each $\hat{Q}_i$ with (k-1) specified c_p^2-values (see card type 5) which define k mutually exclusive and exhaustive areas between coaxial contour ellipses. If $c^2_{p_{j-1}} \le \hat{Q}_i < c^2_{p_j}$ $(j=1,2,\cdots,k)$, the i^{th} observation (x_{1i},x_{2i}) falls within the

j^{th} contour ellipse and outside the $(j-1)^{th}$ contour ellipse. The number of (x_{1i},x_{2i})-observations, $0_j(j=1,2,\cdots,k)$, falling within each of the k areas is then determined, and X^2 as given by equation (8) is computed.

If the quantity (k-6) > 0, the statement "CHI-SQUARE = X^2 WITH (k-6) DEGREES OF FREEDOM" is printed (see page 19); if the quantity (k-6) ≤ 0, the statement "CHI-SQUARE COULD NOT BE COMPUTED BECAUSE OF INSUFFICIENT NUMBER OF INTERVALS" is printed.

III. INPUT PREPARATION

A. Deck Setup

The input deck is listed below by card type. Multiple jobs may be processed by stacking sets of card types 1 through 10. Up to twenty runs may be performed for each set of sample data by stacking sets of card types 4 through 9 behind card type 10.

CARD TYPE 1 - JOB IDENTIFICATION CARD

CARD TYPE 2 - VARIABLE FORMAT CARD

CARD TYPE 3 - NUMBER OF RUNS CARD

CARD TYPE 4 - MAIN CONTROL CARD

CARD TYPE 5 - INTERVAL IDENTIFICATION CARD

CARD TYPE 6 - CONTOUR IDENTIFICATION CARD

CARD TYPE 7 - PLOT HEADING CARD (optional)

CARD TYPE 8 - x_1 TRANSFORMATION CONSTANT CARD

CARD TYPE 9 - x_2 TRANSFORMATION CONSTANT CARD

CARD TYPE 10- SAMPLE DATA CARD

B. Input Deck Description

CARD TYPE 1 - JOB IDENTIFICATION CARD

Column	Format	Program Variable	Explanation
1-80	10A8	JØB(1)-JØB(10)	For job identification.

CARD TYPE 2 - VARIABLE FORMAT CARD

Column	Format	Program Variable	Explanation
1-80	10A8	FMT(1)-FMT(10)	The format by which the (x_1,x_2)-values (card type 10 or input record if data is on tape) are to be read. The format specifications must be enclosed by parentheses.

CARD TYPE 3 - NUMBER OF RUNS CARD

Column	Format	Program Variable	Explanation
4-5	I2	NRUN	The number of runs to be performed per set of sample data. $1 \leq$ NRUN ≤ 20.

CARD TYPE 4 - MAIN CONTROL CARD

Column	Format	Program Variable	Explanation
5	I1	NPRINT	0 or blank - Categorization is not to be printed. 1 - Categorization is to be printed.
10	I1	JPLØT	0 or blank - No plotting is to be performed. 1 - Both data and contours are to be plotted. 2 - Data only is to be plotted.
14-15	I2	NØTIC	The number of desired tick marks on the axis corresponding to the larger range of x_1 and x_2. If NØTIC = 0 or blank, NØTIC is set equal to 15.

Column	Format	Program Variable	Explanation
20	I1	INPUT	0 or blank - Sample data is on punched cards. 1 - Sample data is on tape.
25	I1	NTYPES	0 or blank - Plot headings (card type 7) are not to be input. 1 - Plot headings are to be input.
29-30	I2	ITRAN1	Number corresponding to the transformation to be performed on the x_1-values. $1 \leq$ ITRAN1 ≤ 13.
34-35	I2	ITRAN2	Number corresponding to the transformation to be performed on the x_2-values. $1 \leq$ ITRAN2 ≤ 13.
39-40	I2	IRA	The number of decimal digits to be used in labeling abscissa tick marks. $0 \leq$ IRA ≤ 14.
44-45	I2	IRØ	The number of decimal digits to be used in labeling ordinate tick marks. $0 \leq$ IRØ ≤ 14.
49-50	I2	NPAIR	The number of pairs of (x_1, x_2)-values per sample data card or sample data input record if data is on tape.
55	I1	NN	The number of sets of probability values to be used to define interval bounds for the Chi-Square Test. $1 \leq$ NN ≤ 5.
60	I1	NL	The number of sets of probability values to be used to define contours to be plotted. $1 \leq$ NL ≤ 5.

TRANSFORMATIONS

The following numbers are used to identify the thirteen available transformations (ITRAN1 and ITRAN2).

Transformation Number	Transformation
1	$x \leftarrow x$
2	$x \leftarrow \ln x$
3	$x \leftarrow \ln(\ln x)$
4	$x \leftarrow \ln(A+x)$
5	$x \leftarrow \ln(B+\ln(C+x))$
6	$x \leftarrow \sqrt{x}$
7	$x \leftarrow 1/x$
8	$x \leftarrow 1/(D+x)$
9	$x \leftarrow \sin^{-1}x$
10	$x \leftarrow 2 \sin^{-1}\sqrt{x}$
11	$x \leftarrow x/E$
12	$x \leftarrow \sin x$
13	$x \leftarrow \cos x$

Constants A,B,C,D, and E are input on card type 8 for x_1 and card type 9 for x_2.

CARD TYPE 5 - INTERVAL IDENTIFICATION CARD

Column	Format	Program Variable	Explanation
1-5	F5.3	PS(1)	The smallest probability value for the 1st set of probability values defining interval bounds for the Chi-Square Test.
6-10	F5.3	PDELT(1)	The increment value for the 1st set of probability values defining interval bounds for the Chi-Square Test.
11-15	F5.3	PE(1)	The largest probability value for the 1st set of probability values defining interval bounds for the Chi-Square Test.
16-20	F5.3	PS(2)	"
21-25	F5.3	PDELT(2)	"
26-30	F5.3	PE(2)	"
31-35	F5.3	PS(3)	"
36-40	F5.3	PDELT(3)	"
41-45	F5.3	PE(3)	"
46-50	F5.3	PS(4)	"
51-55	F5.3	PDELT(4)	"
56-60	F5.3	PE(4)	"
61-65	F5.3	PS(5)	The smallest, increment, and largest probability value for the 5th set of probability values defining interval bounds for the Chi-Square Test.
66-70	F5.3	PDELT(5)	
71-75	F5.3	PE(5)	

CARD TYPE 6 - CONTOUR IDENTIFICATION CARD

This card type is omitted if JPLØT = 0 or JPLØT = 2 on card type 4.

Column	Format	Program Variable	Explanation
1-5	F5.3	PPS(1)	The smallest probability value for the 1st set of probability values defining contours to be plotted.

Column	Format	Program Variable	Explanation
6-10	F5.3	PPDELT(1)	The increment value for the 1st set of probability values defining contours to be plotted.
11-15	F5.3	PPE(1)	The largest probability value for the 1st set of probability values defining contours to be plotted.
16-20	F5.3	PPS(2)	"
21-25	F5.3	PPDELT(2)	"
26-30	F5.3	PPE(2)	"
31-35	F5.3	PPS(3)	"
36-40	F5.3	PPDELT(3)	"
41-45	F5.3	PPE(3)	"
46-50	F5.3	PPS(4)	"
51-55	F5.3	PPDELT(4)	"
56-60	F5.3	PPE(4)	"
61-65	F5.3	PPS(5)	The smallest, increment, and largest probability value for the 5th set of probability values defining contours to be plotted.
66-70	F5.3	PPDELT(5)	
71-75	F5.3	PPE(5)	

CARD TYPE 7 - PLOT HEADING CARD

This card type is omitted if NTYPES = 0 on card type 4. Two card types 7 are used for identification.

Column	Format	Program Variable	Explanation
1-80	10A8	XID1(1) - XID1(10)	The first line of the plot identification.
1-80	10A8	XID2(1) - XID2(10)	The second line of the plot identification.

CARD TYPE 8 - x, TRANSFORMATION CONSTANT CARD

This card type is omitted if ITRAN1 = 1 on card type 4. If $2 \leq$ ITRAN1 ≤ 13, but the transformation corresponding to the transformation

number does not contain one of the constants A,B,C,D, or E, a blank card must replace card type 8.

Column	Format	Program Variable	Explanation
1-14	E14.8	ATRAN1	The constant A in transformation number 4 for transforming x_1-values.
15-28	E14.8	BTRAN1	The constant B in transformation number 5 for transforming x_1-values.
29-42	E14.8	CTRAN1	The constant C in transformation number 5 for transforming x_1-values.
43-56	E14.8	DTRAN1	The constant D in transformation number 8 for transforming x_1-values.
57-70	E14.8	ETRAN1	The constant E in transformation number 11 for transforming x_1-values.

CARD TYPE 9 - x_2 TRANSFORMATION CONSTANT CARD

This card type is omitted if ITRAN2 = 1 on card type 4. If $2 \leq$ ITRAN2 ≤ 13, but the transformation corresponding to the transformation number does not contain one of the constants A,B,C,D, or E, a blank card must replace card type 9.

Column	Format	Program Variable	Explanation
1-14	E14.8	ATRAN2	The constant A in transformation number 4 for transforming x_2-values.
15-28	E14.8	BTRAN2	The constant B in transformation number 5 for transforming x_2-values.
29-42	E14.8	CTRAN2	The constant C in transformation number 5 for transforming x_2-values.
43-56	E14.8	DTRAN2	The constant D in transformation number 8 for transforming x_2-values.
57-70	E14.8	ETRAN2	The constant E in transformation number 11 for transforming x_2-values.

CARD TYPE 10 - SAMPLE DATA CARD

Column	Format	Program Variable	Explanation
Variable	Specified on card type 2	IEND	Indicates termination of the sample data cards.*
Variable	Specified on card type 2	X1(I) & X2(I)	The i^{th} pair of (x_{1i},x_{2i})-values; $i \leq 4,000$.

*If IEND = 0 or blank, NPAIR (x_1,x_2)-values are on the card and at least one more sample data card follows.

If IEND > 0, IEND (x_1,x_2)-values are on the card and this card is the last sample data card.

C. Request Sheets

If the sample data (card type 10) is input on punched cards, the job request sheet is prepared as shown below.

SECURITY CLASSIFICATION	NAME	IDENT. NO.	ROOM	BLDG.	PHONE	
☐ TS ☐ S. ☐ C. ☒ U.	J. BROWN	A3	A141	1200	7309	SETUP #____ OF____

☐ COMPILE ☒ GO ☐ COMPILGO	☐ CK OUT ☒ PROC	JOB CARD: CHARGE CODE	IDENT.	PROGRAMMER	JOB TITLE	EST. COMPILER TIME	EST. EXECUTION TIME	DATE
		2158	BC	A3	BI - CHI		30 SEC.	10/20/67

TAPES CALLED FOR BY PROBLEM PROGRAM

TAPE NUMBER	Scratch									
FILE PROTECT ON										
PROGRAMMER NUMBER										
SPECIAL HANDLING (See attached Inst.)										

OPERATOR'S COMMENTS

☐ ABEOJ ☐ HOLD ☐ LOOP ☐ IF CHECKED, SEE REVERSE SIDE FOR ADDITIONAL COMMENTS.

SPECIAL INSTRUCTIONS (continued on reverse)

7030 JOB REQUEST NDW-NWL-5230-29 (REV. 11-66)

If the sample data is input on tape, the tape number is written on the job request sheet instead of "Scratch" and the tape number is punched in place of XXXX on the "REEL, PULXXXX" card (second card) of the IOD deck.

If plots are requested (JPLØT = 1 or 2 on card type 4), the CRT request sheet is prepared as shown below.

TRAID CAMERA OUTPUT FILM OR PAPER COPIES REQUEST
NDW-NWL-5600 2 (REV. 5-66)

PROGRAMMER	ROOM	BLDG.	PHONE	DATE
J. BROWN	A141	1200	7309	10/20/67

FILM IDENTIFICATION

A3-BC

APPROXIMATE NUMBER OF FRAMES

The total number of plots requested for jobs being processed.

NUMBER OF PAPER COPIES PER FRAME

The number of copies of plots desired.

FOR OPERATORS ONLY

COUNTER READING START	FINISH

DATE AND TIME PROBLEM RAN

IV. OUTPUT FORMAT

A. System Output

CONTROL CARDS

(Job identification as given on card type 1)

VARIABLE FORMAT CARD _____

NRUN = _____

IRUN = _____

NPRINT = _____ JPLOT = _____ MOTIC = _____ INPUT = _____ NTYPES = _____ ITRAN1 = _____

ITRAN2 = _____ IRA = _____ IRO = _____ NPAIR = _____ NN = _____ NL = _____

P-VALUES IDENTIFICATION FOR CHI-SQUARE TEST
(Identification as given on card type 5)

P-VALUES IDENTIFICATION FOR PLOTS
(Identification as given on card type 6)

TRANSFORMATION CODE FOR X1 VALUES IS _____
TRANSFORMATION CONSTANTS FOR X1 VALUES ARE-
ATRAN1 = _____ BTRAN1 = _____ CTRAN1 = _____ DTRAN1 = _____ ETRAN1 = _____

TRANSFORMATION CODE FOR X2 VALUES IS _____
TRANSFORMATION CONSTANTS FOR X2 VALUES ARE-
ATRAN2 = _____ BTRAN2 = _____ CTRAN2 = _____ DTRAN2 = _____ ETRAN2 = _____

	ORIGINAL DATA		TRANSFORMED DATA	
I	X1	X2	X1	X2
1	x_{11}	x_{21}	x'_{11}	x'_{21}
2	x_{12}	x_{22}	x'_{12}	x'_{22}
:	:	:	:	:
:	:	:	:	:
:	:	:	:	:
:	:	:	:	:
i	x_{1i}	x_{2i}	x'_{1i}	x'_{2i}
:	:	:	:	:
:	:	:	:	:
:	:	:	:	:
:	:	:	:	:
n	x_{1n}	x_{2n}	x'_{1n}	x'_{2n}

NOTE: If ITRAN1 and ITRAN2 equal 1, the transformed and original are identical; i.e., $x'_{vi} \equiv x_{vi}$; $v=1,2$; $i=1,2,\cdots,n$.

MIN X1 = $x_{1(1)}$ MAX X1 = $x_{1(n)}$

MIN X2 = $x_{2(1)}$ MAX X2 = $x_{2(n)}$

R = r MEAN X1 = $\bar{x}_1$ MEAN X2 = $\bar{x}_2$ VARIANCE X1 = s_1^2 VARIANCE X2 = s_2^2

STAN DEV X1 = s_1 STAN DEV X2 = s_2

X2 = a_1 + b_1 *X1

X1 = a_2 + b_2 *X2

C2(1) = $c^2_{p_1}$ P(1) = p_1

(all i; x_{1i}, x_{2i}; $\hat{Q}_i$ for which $0 \le \hat{Q}_i < c^2_{p_1}$)

C2(2) = $c^2_{p_2}$ P(2) = p_2

(all i; x_{1i}, x_{2i}; $\hat{Q}_i$ for which $c^2_{p_1} \le \hat{Q}_i < c^2_{p_2}$)

:
:
:

C2(J) = $c^2_{p_j}$ P(J) = p_j

(all i; x_{1i}, x_{2i}; $\hat{Q}_i$ for which $c^2_{p_{j-1}} \le \hat{Q}_i < c^2_{p_j}$)

:
:
:

C2(K-1) = $c^2_{p_{k-1}}$ P(K-1) = p_{k-1}

(all i; x_{1i}, x_{2i}; $\hat{Q}_i$ for which $c^2_{p_{k-2}} \le \hat{Q}_i < c^2_{p_{k-1}}$)

C2(K) = INFINITY P(K) = 1.000

(all i; x_{1i}, x_{2i}; $\hat{Q}_i$ for which $c^2_{p_{k-1}} \le \hat{Q}_i < \infty$)

NOTE: The categorization illustrated on this page is optional output and is obtained only when NPRINT = 1 (see card type 4).

CHI-SQUARE TEST

RUN NUMBER __i__ WITH __n__ OBSERVATIONS AND __k__ INTERVALS

INTERVAL INDEX(J)	UPPER BOUND C2(J)	PROBABILITY P(J)	OBS FREQ	THEO FREQ	CHI-SQU CONTR
1	$c^2_{p_1}$	p_1	O_1	$\hat{E}_1$	$(O_1-\hat{E}_1)^2/\hat{E}_1$
2	$c^2_{p_2}$	p_2	O_2	$\hat{E}_2$	$(O_2-\hat{E}_2)^2/\hat{E}_2$
:	:	:	:	:	:
:	:	:	:	:	:
j	$c^2_{p_j}$	p_j	O_j	$\hat{E}_j$	$(O_j-\hat{E}_j)^2/\hat{E}_j$
:	:	:	:	:	:
:	:	:	:	:	:
k-1	$c^2_{p_{k-1}}$	p_{k-1}	O_{k-1}	$\hat{E}_{k-1}$	$(O_{k-1}-\hat{E}_{k-1})^2/\hat{E}_{k-1}$
k	∞	1.000	O_k	$\hat{E}_k$	$(O_k-\hat{E}_k)^2/\hat{E}_k$

CHI-SQUARE = __X^2__ WITH __(k-6)__ DEGREES OF FREEDOM

B. CRT Output

The plots are performed by the system "GRF Plot Subroutine" and are plotted by the CRT printer units. The plots are in an ordinary Cartesian coordinate system, and the origin is at $(x_{1(1)}, x_{2(1)})$. The abscissa of the system is the x_1-axis; the ordinate of the system is the x_2-axis. An asterisk (*) is used to denote an (x_{1i}, x_{2i})-observation; a zero (0) is used to denote the center $(\bar{x}_1, \bar{x}_2)$ of the distribution of the observations. In addition, a zero (0) is used to denote points on each of the estimated regression lines, $x_2 = f(x_1)$ and $x_1 = f(x_2)$. Up to four points (depending upon the range of the particular plot) are plotted for each regression line. The plotted points on the regression lines, however, are <u>not</u> connected.

C. Optional Output

The output illustrated on pages 15, 16, 17, and 19 of Section IV.A is automatically printed for each run processed. The categorization (page 18) and the CRT output is controlled by the analyst. This flexibility is provided by the program variables, NPRINT and JPLØT, on card type 4. The allowable input values (two for NPRINT and three for JPLØT) for these program variables permit six different output options. A quick reference of the available output options is given in the following table.

NPRINT	JPLØT	OUTPUT
0	0	1. Chi-Square Test
0	1	1. Chi-Square Test 2. Plot of data with contours
0	2	1. Chi-Square Test 2. Plot of data
1	0	1. Chi-Square Test 2. Categorization
1	1	1. Chi-Square Test 2. Categorization 3. Plot of data with contours
1	2	1. Chi-Square Test 2. Categorization 3. Plot of data

D. Program Running Time

The running time is printed at the end of each run processed. The following table of observed times may be used as a guide for estimating program running times.

Sample Size	No. of Intervals	Categorization	Plots	Time in Seconds
50	10	Yes	Data & Contours	8.2
100	14	No	Data & Contours	9.8
150	12	Yes	Data & Contours	14.3
500	100	No	No	16.2
1000	150	Yes	Data & Contours	56.5
2500	200	No	Data & Contours	81.2

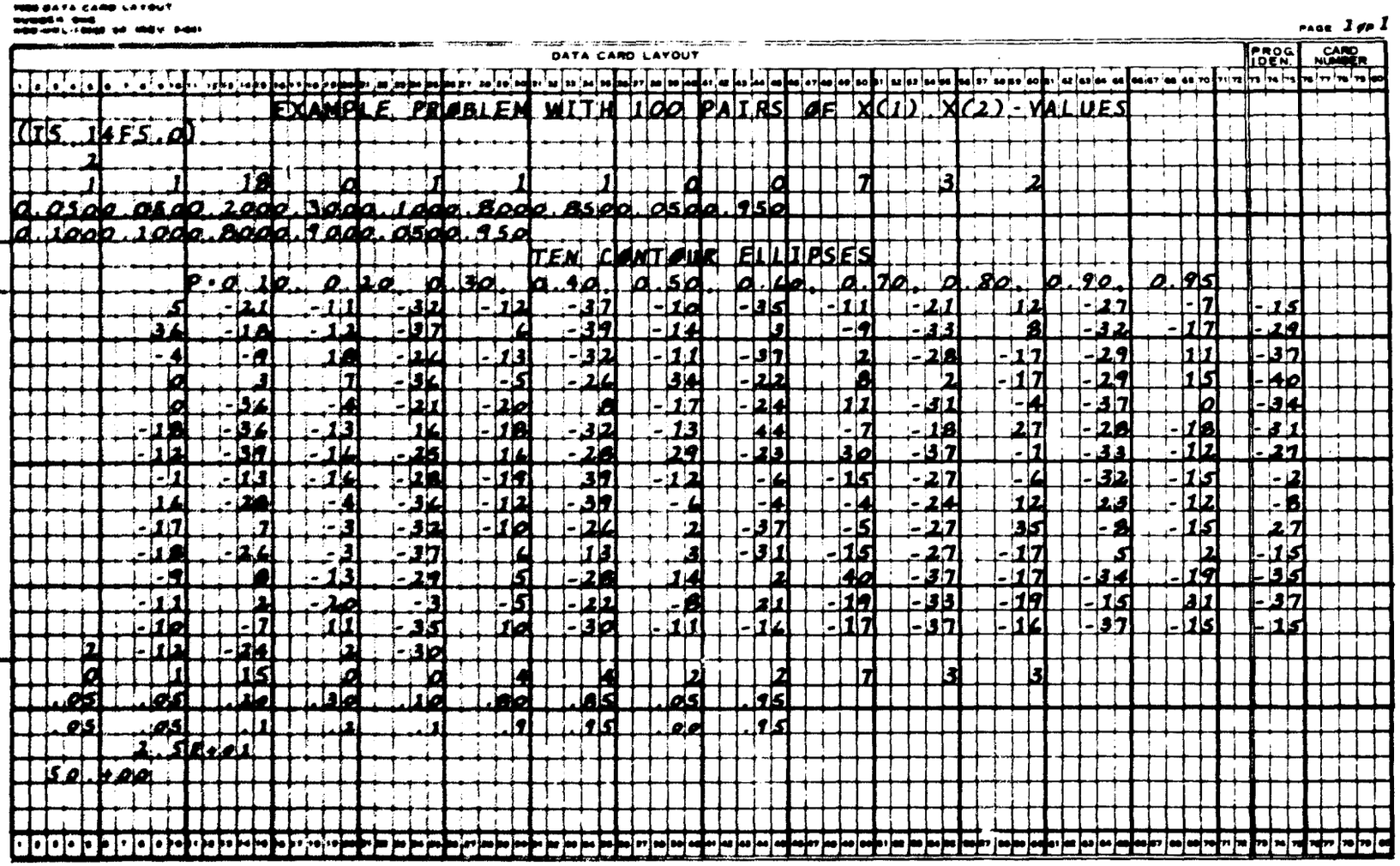
V. NUMERICAL EXAMPLE

A. Problem Description

The example problem consists of 100 observations (pairs of (x_1, x_2)-values) randomly sampled from a continuous two-dimensional distribution. Two runs were performed, one on the original observations and one on the logarithmically transformed values. Because some of the original observations (both x_1-values and x_2-values) were negative, transformation number 4 was used. For transforming x_1, the constant A was set equal to 25; for transforming x_2, A was set equal to 50. In both runs, 14 intervals were specified for the Chi-Square Test. Categorization was requested for the first run, but it was not requested for the second run. The heading of the plot (card type 7) was input for the first run only.

The input deck setup is illustrated on the following Data Card Layout Sheet.

B. Input for Example Problem



C. Program Output

```
                                                        CONTROL CARDS
              EXAMPLE PROBLEM WITH 100 PAIRS OF X(1),X(2)-VALUES

 VARIABLE FORMAT CARD      (F5.1,4F5.0)

NRUN =  2

IRUN =  1

NPRINT = 1      JPLOT = 1      NOTIC = 18      INPUT = 0      NTYPES =1      ITRAN1 =  1

ITRAN2 =  1     IRA = 0        IRO = 0         NPAIR =  7     NN = 3          NL = 2

P-VALUES IDENTIFICATION FOR CHI-SQUARE TEST
     0.050(0.050(0.200  0.300(0.100(0.800  0.950(0.050(0.950

P-VALUES IDENTIFICATION FOR PLOTS
     0.100(0.100(0.600  0.900(0.050(0.950
```

	ORIGINAL DATA		TRANSFORMED DATA	
I	X1	X2	X1	X2
1	.5000E+01	-.2100E+02	.50E+01	-.2100E+02
2	-.1100E+02	-.3200E+02	-.11E+02	-.3200E+02
3	-.1200E+02	-.3700E+02	-.12E+02	-.3700E+02
4	-.1000E+02	-.3500E+02	-.10E+02	-.3500E+02
5	-.1100E+02	-.2100E+02	-.11E+02	-.2100E+02
6	.1200E+02	-.2700E+02	.12E+02	-.2700E+02
7	-.7000E+01	-.1500E+02	-.70E+01	-.1500E+02
8	.3600E+02	-.1800E+02	.36E+02	-.1800E+02
9	-.1200E+02	-.3700E+02	-.12E+02	-.3700E+02
10	.6000E+01	-.3900E+02	.60E+01	-.3900E+02
11	-.1400E+02	.3000E+01	-.14E+02	.3000E+01
12	-.9000E+01	-.3300E+02	-.90E+01	-.3300E+02
13	.8000E+01	-.3200E+02	.80E+01	-.3200E+02
14	-.1700E+02	-.2900E+02	-.17E+02	-.2900E+02
15	-.4000E+01	-.9000E+01	-.40E+01	-.9000E+01
16	.1800E+02	-.2600E+02	.18E+02	-.2600E+02
17	-.1300E+02	-.3200E+02	-.13E+02	-.3200E+02
18	-.1100E+02	-.3700E+02	-.11E+02	-.3700E+02
19	.2000E+01	-.2800E+02	.20E+01	-.2800E+02
20	-.1700E+02	-.2900E+02	-.17E+02	-.2900E+02
21	.1100E+02	-.3700E+02	.11E+02	-.3700E+02
22	.0000E 00	.3000E+01	.00E+00	.3000E+01
23	.7000E+01	-.3600E+02	.70E+01	-.3600E+02
24	-.5000E+01	-.2600E+02	-.50E+01	-.2600E+02
25	.3400E+02	-.2200E+02	.34E+02	-.2200E+02
26	.8000E+01	.2000E+01	.80E+01	.2000E+01
27	-.1700E+02	-.2900E+02	-.17E+02	-.2900E+02
28	.1500E+02	-.4000E+02	.15E+02	-.4000E+02
29	.0000E 00	-.3600E+02	.00E+00	-.3600E+02
30	-.4000E+01	-.2100E+02	-.40E+01	-.2100E+02
31	-.2000E+02	.8000E+01	-.20E+02	.8000E+01
32	-.1700E+02	-.2400E+02	-.17E+02	-.2400E+02
33	.1100E+02	-.3100E+02	.11E+02	-.3100E+02
34	-.4000E+01	-.3700E+02	-.40E+01	-.3700E+02
35	.0000E 00	-.3400E+02	.00E+00	-.3400E+02
36	-.1800E+02	-.3600E+02	-.18E+02	-.3600E+02
37	-.1300E+02	.1600E+02	-.13E+02	.1600E+02
38	-.1800E+02	-.3200E+02	-.18E+02	-.3200E+02
39	-.1300E+02	.4400E+02	-.13E+02	.4400E+02
40	-.7000E+01	-.1800E+02	-.70E+01	-.1800E+02
41	.2700E+02	-.2800E+02	.27E+02	-.2800E+02
42	-.1800E+02	-.3100E+02	-.18E+02	-.3100E+02
43	-.1200E+02	-.3900E+02	-.12E+02	-.3900E+02
44	-.1600E+02	-.2500E+02	-.16E+02	-.2500E+02
45	.1600E+02	-.2800E+02	.16E+02	-.2800E+02
46	.2900E+02	-.2300E+02	.29E+02	-.2300E+02
47	.3000E+02	-.3700E+02	.30E+02	-.3700E+02
48	-.1100E+01	-.3300E+02	-.11E+01	-.3300E+02
49	-.1200E+02	-.2700E+02	-.12E+02	-.2700E+02
50	-.1000E+01	-.1300E+02	-.10E+01	-.1300E+02

51	-.16000000E+02	-.28000000E+02	-.16000000E+02	-.28000000E+02
52	-.19000000E+02	.39000000E+02	-.19000000E+02	.39000000E+02
53	-.12000000E+02	-.60000000E+01	-.12000000E+02	-.60000000E+01
54	-.15000000E+02	-.27000000E+02	-.15000000E+02	-.27000000E+02
55	-.50000000E+01	-.32000000E+02	-.50000000E+01	-.32000000E+02
56	-.15000000E+02	-.20000000E+01	-.15000000E+02	-.20000000E+01
57	.16000000E+02	-.28000000E+02	.16000000E+02	-.28000000E+02
58	-.40000000E+01	-.36000000E+02	-.40000000E+01	-.36000000E+02
59	-.12000000E+02	-.39000000E+02	-.12000000E+02	-.39000000E+02
60	-.60000000E+01	-.40000000E+01	-.60000000E+01	-.40000000E+01
61	-.40000000E+01	-.24000000E+02	-.40000000E+01	-.24000000E+02
62	.12000000E+02	.23000000E+02	.12000000E+02	.23000000E+02
63	-.12000000E+02	-.80000000E+01	-.12000000E+02	-.80000000E+01
64	-.17000000E+02	.70000000E+01	-.17000000E+02	.70000000E+01
65	-.30000000E+01	-.32000000E+02	-.30000000E+01	-.32000000E+02
66	-.10000000E+02	-.26000000E+02	-.10000000E+02	-.26000000E+02
67	.20000000E+01	-.37000000E+02	.20000000E+01	-.37000000E+02
68	-.50000000E+01	-.27000000E+03	-.50000000E+01	-.27000000E+02
69	.35000000E+02	-.80000000E+01	.35000000E+02	-.80000000E+01
70	-.15000000E+02	.27000000E+02	-.15000000E+02	.27000000E+02
71	-.18000000E+02	-.26000000E+02	-.18000000E+02	-.26000000E+02
72	-.30000000E+01	-.37000000E+02	-.30000000E+01	-.37000000E+02
73	.60000000E+01	.13000000E+02	.60000000E+01	.13000000E+02
74	.30000000E+01	-.31000000E+02	.30000000E+01	-.31000000E+02
75	-.15000000E+02	-.27000000E+02	-.15000000E+02	-.27000000E+02
76	-.17000000E+02	.50000000E+01	-.17000000E+02	.50000000E+01
77	.20000000E+01	-.15000000E+02	.20000000E+01	-.15000000E+02
78	-.90000000E+01	.80000000E+01	-.90000000E+01	.80000000E+01
79	-.13000000E+02	-.29000000E+02	-.13000000E+02	-.29000000E+02
80	.50000000E+01	-.28000000E+02	.50000000E+01	-.28000000E+02
81	.14000000E+02	.20000000E+01	.14000000E+02	.20000000E+01
82	.40000000E+02	-.37000000E+02	.40000000E+02	-.37000000E+02
83	-.17000000E+02	-.34000000E+02	-.17000000E+02	-.34000000E+02
84	-.19000000E+02	-.35000000E+02	-.19000000E+02	-.35000000E+02
85	-.11000000E+02	.20000000E+01	-.11000000E+02	.20000000E+01
86	-.20000000E+02	-.30000000E+01	-.20000000E+02	-.30000000E+01
87	-.50000000E+01	-.22000000E+02	-.50000000E+01	-.22000000E+02
88	-.80000000E+01	.21000000E+02	-.80000000E+01	.21000000E+02
89	-.19000000E+02	-.33000000E+02	-.19000000E+02	-.33000000E+02
90	-.19000000E+02	-.15000000E+02	-.19000000E+02	-.15000000E+02
91	.31000000E+02	-.37000000E+02	.31000000E+02	-.37000000E+02
92	-.10000000E+02	-.70000000E+01	-.10000000E+02	-.70000000E+01
93	.11000000E+02	-.35000000E+02	.11000000E+02	-.35000000E+02
94	.10000000E+02	-.30000000E+02	.10000000E+02	-.30000000E+02
95	-.11000000E+02	-.16000000E+02	-.11000000E+02	-.16000000E+02
96	-.17000000E+02	-.37000000E+02	-.17000000E+02	-.37000000E+02
97	-.16000000E+02	-.37000000E+02	-.16000000E+02	-.37000000E+02
98	-.15000000E+02	-.15000000E+02	-.15000000E+02	-.15000000E+02
99	-.12000000E+02	-.24000000E+02	-.12000000E+02	-.24000000E+02
100	.20000000E+01	-.30000000E+02	.20000000E+01	-.30000000E+02

MIN X1 =-.20600000E+02 MAX X1 = .40000000E+02
MIN X2 =-.42000000E+02 MAX X2 = .44000000E+02

R=-.15631697E-00 MEAN X1=-.12500000E+01 MEAN X2=-.20610000E+02 VARIANCE X1= .21443182E+03 VARIANCE X2= .33411909E+03
STAN DEV X1= .14643691E+02 STAN DEV X2= .18278925E+02

X2=-.21244155E+02 + -.19512454E-00 *X1
X1=-.59309395E+01 + -.12522753E-00 *X2

```
C2(  1) =  .10258659E-00      P(  1) =     0.050
  30-(-.430U0000E+01,-.21000000E+02)-  .35057392E-02
  40-(-.730U0000E+01,-.18000000E+02)-  .76403707E-01
  61-(-.430U0000E+01,-.24000000E+02)-  .40989637E-01
  87-(-.530U0000E+01,-.22000000E+02)-  .23479463E-01
C2(  2) =  .21C72103E-00      P(  2) =     0.100
   7-(-.700U0000E+01,-.15000000E+02)-  .13858930E-00
  24-(-.530U0000E+01,-.26000000E+02)-  .11506190E-00
  68-(-.530U0000E+01,-.27000000E+02)-  .15329709E-00
C2(  3) =  .32503786E-00      P(  3) =     0.150
   1-(  .530U0000E+01,-.21000000E+02)-  .32157319E-00
   5-(-.110U0000E+02,-.21000000E+02)-  .29120168E-00
  19-(  .200U0000E+01,-.28000000E+02)-  .25285154E-00
  53-(-.100U0000E+01,-.13000000E+02)-  .22236918E-00
  77-(  .200U0000E+01,-.15000000E+02)-  .26357234E-00
  95-(-.110U0000E+02,-.16000000E+02)-  .30954118E-00
C2(  4) =  .44628719E-00      P(  4) =     0.200
  15-(-.430U0000E+01,-.90000000E+01)-  .40570378E-00
  65-(-.300U0000E+01,-.32000000E+02)-  .39489584E-00
  66-(-.100U0000E+02,-.26000000E+02)-  .35049021E-00
  74-(  .300U0000E+01,-.31000000E+02)-  .44017110E-00
  80-(  .500U0000E+01,-.28000000E+02)-  .41991641E-00
  99-(-.120U0000E+02,-.24000000E+02)-  .43476131E-00
 100-(  .200U0000E+01,-.30000000E+02)-  .34329945E-00
C2(  5) =  .71334989E+00      P(  5) =     0.300
  35-(  .000U0000E 00,-.34000000E+02)-  .54844244E+00
  48-(-.100U0000E+01,-.37000000E+02)-  .44178510E-00
  49-(-.120U0000E+02,-.27000000E+02)-  .55820154E+00
  55-(-.600U0000E+01,-.32000000E+02)-  .47165807E-00
  92-(-.100U0000E+02,-.70000000E+01)-  .47608911E+00
  98-(-.150U0000E+02,-.15000000E+02)-  .67761919E+00
C2(  6) =  .10216512E+01      P(  6) =     0.400
   2-(-.110U0000E+02,-.32000000E+02)-  .79080678E+00
   4-(-.130U0000E+02,-.35000000E+02)-  .96937224E+00
  12-(-.900U0000E+01,-.33000000E+02)-  .71430460E+00
  13-(  .800U0000E+01,-.32000000E+02)-  .84959971E+00
  17-(-.130U0000E+02,-.32000000E+02)-  .98539038E+00
  29-(  .000U0000E 00,-.36000000E+02)-  .71724917E+00
  32-(-.170U0000E+02,-.24000000E+02)-  .99483768E+00
  34-(-.400U0000E+01,-.37000000E+02)-  .84194499E+00
  44-(-.160U0000E+02,-.25000000E+02)-  .90323446E+00
  53-(-.120U0000E+02,-.60000000E+01)-  .86778958E+00
  54-(-.150U0000E+02,-.27000000E+02)-  .87514084E+00
  58-(-.400U0000E+01,-.36000000E+02)-  .74314888E+00
  60-(-.600U0000E+01,-.40000000E+01)-  .82787592E+00
  63-(-.120U0000E+02,-.80000000E+01)-  .72172444E+00
  67-(  .200U0000E+01,-.37000000E+02)-  .85287543E+00
  72-(-.300U0000E+01,-.37000000E+02)-  .81953190E+00
  75-(-.150U0000E+02,-.27000000E+02)-  .87514084E+00
  79-(-.130U0000E+02,-.29000000E+02)-  .76832113E+00
```

```
 94-( .100v0000E+02,-.30000000E+02)- .96078554E+00
C2( 7) = .13862944E+01      P( 7) =    0.500
  3-(-.120v0000E+02,-.37000000E+02)- .13618305E+01
  6-( .120v0000E+02,-.27000000E+02)- .11203172E+01
  9-(-.120v0000E+02,-.37000000E+02)- .13618305E+01
 12-( .600v0000E+01,-.39000000E+02)- .12628955E+01
 14-(-.170v0000E+02,-.29000000E+02)- .12578484E+01
 18-(-.110v0000E+02,-.37000000E+02)- .17623327E+01
 20-(-.170v0000E+02,-.29000000E+02)- .12578484E+01
 23-( .700v0000E+01,-.36000000E+02)- .10400073E+01
 27-(-.170v0000E+02,-.29000000E+02)- .12578484E+01
 33-( .110v0000E+02,-.31000000E+02)- .11246239E+01
 51-(-.160v0000E+02,-.28000000E+02)- .10574492E+01
 71-(-.190v0000E+02,-.26000000E+02)- .12243260E+01
 90-(-.190v0000E+02,-.15000000E+02)- .11765792E+01
 93-( .110v0000E+02,-.35000000E+02)- .13647119E+01
C2( 8) = .18325815E+01      P( 8) =    0.600
 21-( .110v0000E+02,-.37000000E+02)- .15152108E+01
 38-(-.180v0000E+02,-.32000000E+02)- .16351596E+01
 42-(-.190v0000E+02,-.31000000E+02)- .15546811E+01
 43-(-.120v0000E+02,-.39000000E+02)- .15961861E+01
 45-( .160v0000E+02,-.28000000E+02)- .17686248E+01
 56-(-.150v0000E+02,-.20000000E+01)- .14606953E+01
 57-( .160v0000E+02,-.29000000E+02)- .17686248E+01
 59-(-.120v0000E+02,-.39000000E+02)- .15961861E+01
 83-(-.170v0000E+02,-.34000000E+02)- .16742545E+01
 85-(-.110v0000E+02, .20000000E+01)- .16456777E+01
C2( 9) = .24079456E+01      P( 9) =    0.700
 11-(-.140v0000E+02, .30000000E+01)- .19587029E+01
 16-( .180v0000E+02,-.26000000E+02)- .21106005E+01
 22-( .000v0000E 00, .30000000E+01)- .18525107E+01
 28-( .150v0000E+02,-.40000000E+02)- .23219173E+01
 36-(-.180v0000E+02,-.36000000E+02)- .20384317E+01
 84-(-.190v0000E+02,-.35000000E+02)- .20924373E+01
 86-(-.200v0000E+02,-.30000000E+01)- .19356173E+01
 89-(-.190v0000E+02,-.33000000E+02)- .18904058E+01
 96-(-.170v0000E+02,-.37000000E+02)- .19977280E+01
 97-(-.160v0000E+02,-.37000000E+02)- .18514277E+01
C2( 10) = .32188758E+01      P( 10) =    0.800
 26-( .800v0000E+01, .20000000E+01)- .24779933E+01
 54-(-.170v0000E+02, .70000000E+01)- .27879579E+01
 75-(-.170v0000E+02, .50000000E+01)- .26943340E+01
 78-(-.900v0000E+01, .80000000E+01)- .24722741E+01
C2( 11) = .37942400E+01      P( 11) =    0.850
 31-(-.210v0000E+02, .80000000E+01)- .32786106E+01
 91-( .140v0000E+02, .20000000E+01)- .34917422E+01
C2( 12) = .46051702E+01      P( 12) =    0.900
 37-(-.130v0000E+02, .16000000E+02)- .41389660E+01
 41-( .270v0000E+02,-.28000000E+02)- .42741702E+01
 73-( .610v0000E+01, .13000000E+02)- .42468380E+01
```

```
C2( 13) =  .59914645E+01      P( 13) =     0.950
  46-( .290u0000E+02,-.23000000E+02)-  .48970471E+01
  47-( .300u0000E+02,-.37000000E+02)-  .54565903E+01
  48-(-.800u0000E+01, .21000000E+02)-  .51829757E+01
  91-( .310u0000E+02,-.37000000E+02)-  .57566372E+01

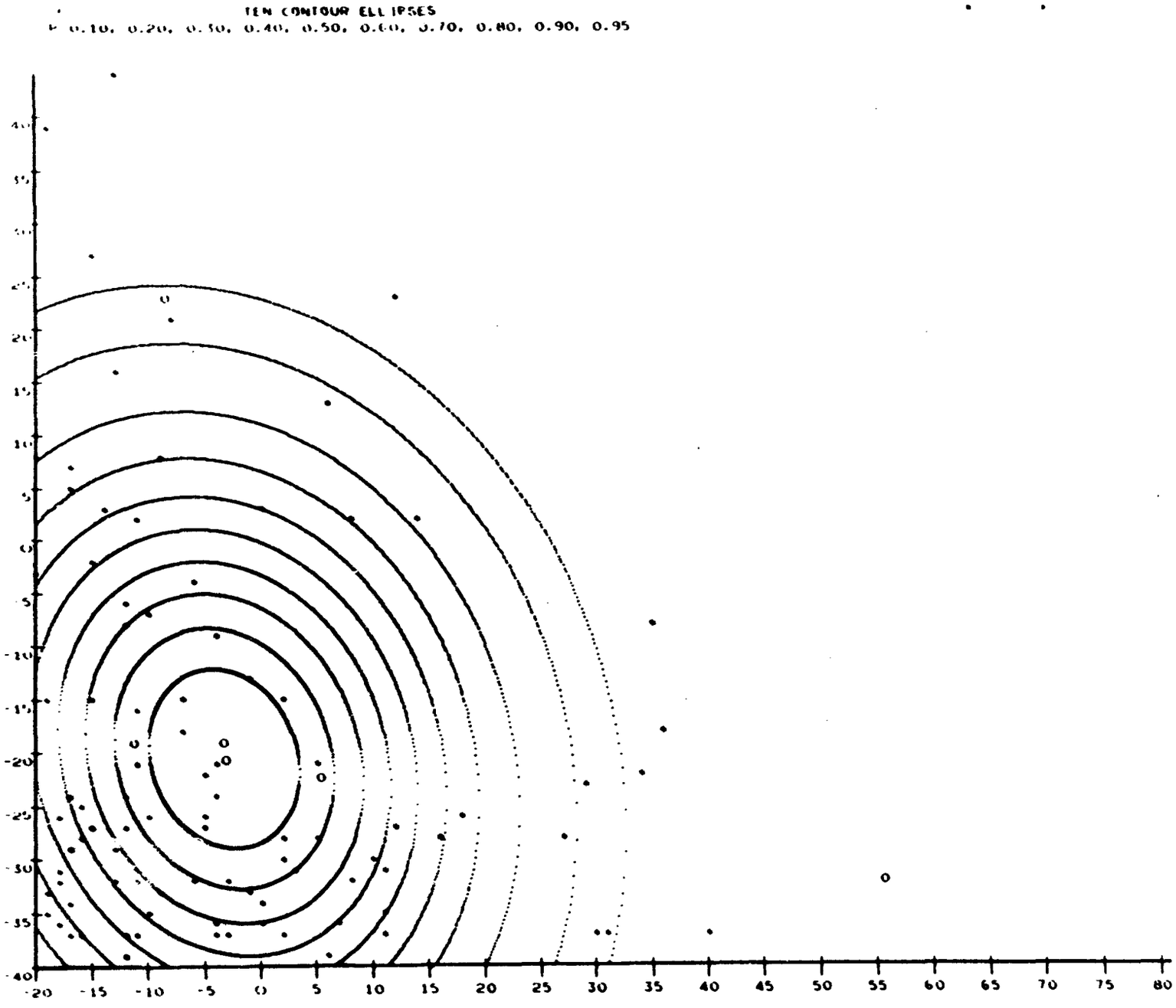
C2( 14) = INFINITY            P( 14) =     1.000
   8-( .360u0000E+02,-.18000000E+02)-  .75078887E+01
  25-( .340u0000E+02,-.22000000E+02)-  .65768927E+01
  39-(-.130u0000E+02, .44000000E+02)-  .12507062E+02
  52-(-.190u0000E+02, .39000000E+02)-  .10963127E+02
  62-( .120u0000E+02, .23000000E+02)-  .77425981E+01
  69-( .350u0000E+02,-.80000000E+01)-  .80591745E+01
  70-(-.150u0000E+02, .27000000E+02)-  .69442840E+01
  82-( .490u0000E+02,-.37000000E+02)-  .89172856E+01
```

CHI-SQUARE TEST

RUN NUMBER 1 WITH 100 OBSERVATIONS AND 14 INTERVALS

INTERVAL INDEX(J)	UPPER BOUND C2(J)	PROBABILITY P(J)	OBS FREQ	THEO FREQ	CHI-SQU CONTR
1	.10258659E-00	0.050	4	5.0000	0.2000
2	.21072103E-00	0.100	3	5.0000	0.8000
3	.32503786E-00	0.150	6	5.0000	0.2000
4	.44628710E-00	0.200	7	5.0000	0.8000
5	.71334989E+00	0.300	6	10.0000	1.6000
6	.10216512E+01	0.400	19	10.0000	8.1000
7	.13862944E+01	0.500	14	10.0000	1.6000
8	.18325815E+01	0.600	10	10.0000	0.0000
9	.24079456E+01	0.700	10	10.0000	0.0000
10	.32188758E+01	0.800	4	10.0000	3.6000
11	.47942400E+01	0.850	2	5.0000	1.8000
12	.66051702E+01	0.900	3	5.0000	0.8000
13	.99914645E+01	0.950	4	5.0000	0.2000
14	9.99999999+614	1.000	8	5.0000	1.8000

CHI-SQUARE = 21.5C000 WITH 8 DEGREES OF FREEDOM


TEN CONTOUR ELLIPSES
P 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 0.95
-20 -15 -10 -5 0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

```
RANGE OVERFLOW
RANGE OVERFLOW
RANGE OVERFLOW
RANGE OVERFLOW
RANGE OVERFLOW
RANGE OVERFLOW
RANGE OVERFLOW
RUNNING TIME IN SECONDS =  .10131937E+02
```

CONTROL CARDS

EXAMPLE PROBLEM WITH 100 PAIRS OF X(1),X(2)-VALUES

VARIABLE FORMAT CARD (I5,14F5.0)

NRUN = 2

IRUN = 2

NPRINT = 0 JPLOT = 1 NOTIC = 15 INPUT = 0 NTYPES =0 ITRAN1 = 4

ITRAN2 = 4 IRA = 2 IRO = 2 NPAIR = 7 NN = 3 NL = 3

P-VALUES IDENTIFICATION FOR CHI-SQUARE TEST
0.050(0.050)0.200 0.300(0.100)0.800 0.850(0.050)0.950

P-VALUES IDENTIFICATION FOR PLOTS
0.050(0.050)0.100 0.200(0.100)0.900 0.950(0.000)0.950

TRANSFORMATION CODE FOR X1 VALUES IS 4
TRANSFORMATION CONSTANTS FOR X1 VALUES ARE-
ATRAN1 = .25030000E+02 BTRAN1 = -.00000000E 00 CTRAN1 = -.00000000E 00 DTRAN1 = -.00000000E 00 ETRAN1 = -.00000000E 00

TRANSFORMATION CODE FOR X2 VALUES IS 4
TRANSFORMATION CONSTANTS FOR X2 VALUES ARE-
ATRAN2 = .50000000E+02 BTRAN2 = -.00000000E 00 CTRAN2 = -.00000000E 00 DTRAN2 = -.00000000E 00 ETRAN2 = -.00000000E 00

I	ORIGINAL DATA X1	X2	TRANSFORMED DATA X1	X2
1	.90000000E+01	-.21000000E+02	.34011974E+01	.33672558E+01
2	-.11000000E+02	-.32000000E+02	.26390573E+01	.28903718E+01
3	-.12000000E+02	-.37000000E+02	.25649494E+01	.25649494E+01
4	-.10000000E+02	-.35000000E+02	.27080502E+01	.27080502E+01
5	-.11000000E+02	-.21000000E+02	.26390573E+01	.33672559E+01
6	.12000000E+02	-.27000000E+02	.36109179E+01	.31354942E+01
7	-.70000000E+01	-.15000000E+02	.28503718E+01	.35553481E+01
8	.96000000E+02	-.18000000E+02	.41108739E+01	.34657359E+01
9	-.12000000E+02	-.37000000E+02	.25649494E+01	.25649494E+01
10	.60000000E+01	-.39000000E+02	.34239872E+01	.23978953E+01
11	-.14000000E+02	.30000000E+01	.23978953E+01	.39702819E+01
12	-.90000000E+01	-.39000000E+02	.27725887E+01	.28332133E+01
13	.80000000E+01	-.32000000E+02	.34565076E+01	.28903718E+01
14	-.17000000E+02	-.29000000E+02	.20794415E+01	.30445224E+01
15	-.40000000E+01	-.90000000E+01	.30445224E+01	.37135721E+01
16	.10000000E+02	-.26000000E+02	.37612001E+01	.31790538E+01
17	-.13000000E+02	-.32000000E+02	.24849066E+01	.28903718E+01
18	-.11000000E+02	-.37000000E+02	.26390573E+01	.25649494E+01
19	.20000000E+01	-.28000000E+02	.32558369E+01	.30910425E+01
20	-.17000000E+02	-.29000000E+02	.20794415E+01	.30445224E+01
21	.11000000E+02	-.37000000E+02	.35835189E+01	.25649494E+01
22	.00000000E 00	.30000000E+01	.32188758E+01	.39702819E+01
23	.70000000E+01	-.36000000E+02	.34657359E+01	.26390573E+01
24	-.50000000E+01	-.26000000E+02	.29557323E+01	.31790538E+01
25	.34000000E+02	-.22000000E+02	.40175374E+01	.33322045E+01
26	.80000000E+01	.20000000E+01	.34565076E+01	.39512437E+01
27	-.17000000E+02	-.29000000E+02	.20794415E+01	.30445224E+01
28	.15000000E+02	-.40000000E+02	.34188795E+01	.23225951E+01
29	.00000000E 00	-.36000000E+02	.32188758E+01	.26390573E+01
30	-.40000000E+01	-.21000000E+02	.30445224E+01	.33672958E+01
31	-.20000000E+02	.80000000E+01	.16094379E+01	.40604430E+01
32	-.17000000E+02	-.24000000E+02	.20794415E+01	.32500965E+01
33	.11000000E+02	-.31000000E+02	.35835189E+01	.29444390E+01
34	-.40000000E+01	-.37000000E+02	.30445224E+01	.25649494E+01
35	.00000000E 00	-.34000000E+02	.32188758E+01	.27725887E+01
36	-.14000000E+02	-.36000000E+02	.19459101E+01	.26390573E+01
37	-.13000000E+02	.16000000E+02	.24849066E+01	.41896547E+01
38	-.18000000E+02	-.32000000E+02	.19459101E+01	.28903718E+01
39	-.13000000E+02	.44000000E+02	.24849066E+01	.45432948E+01
40	-.70000000E+01	-.19000000E+02	.28503718E+01	.34657359E+01
41	.27000000E+02	-.28000000E+02	.33012437E+01	.30910425E+01
42	-.14000000E+02	-.31000000E+02	.19459101E+01	.29444390E+01
43	-.12000000E+02	-.39000000E+02	.25649494E+01	.23978953E+01
44	-.16000000E+02	-.25000000E+02	.21972246E+01	.32188758E+01
45	.14000000E+02	-.28000000E+02	.37135721E+01	.30910425E+01
46	.29000000E+02	-.23000000E+02	.39889840E+01	.32958369E+01
47	.33000000E+02	-.37000000E+02	.40073332E+01	.25649494E+01
48	-.10000000E+01	-.33000000E+02	.31780538E+01	.28332133E+01
49	-.12000000E+02	-.37000000E+02	.25649494E+01	.31354942E+01
50	-.10000000E+01	-.13000000E+02	.31780538E+01	.36109179E+01

91	-.1600\000E+02	-.28000000E+02	.21572246E+01	.30910425E+01
92	-.1900\000E+02	.39000000E+02	.17517595E+01	.44886364E+01
93	-.1200\000E+02	-.60000000E+01	.25649494E+01	.37841896E+01
94	-.1500\000E+02	-.27000000E+02	.23025851E+01	.31354942E+01
95	-.6000\001E+01	-.92000000E+02	.29444390E+01	.28903718E+01
96	-.1500\000E+02	-.20000000E+01	.23025851E+01	.38712010E+01
97	.1600\000E+02	-.28000000E+02	.37135721E+01	.30910425E+01
98	-.4000\000E+01	-.14000000E+02	.30445224E+01	.26390573E+01
99	-.1200\000E+02	-.39000000E+02	.25649494E+01	.23978953E+01
63	-.6000\000E+01	-.40000000E+01	.29444390E+01	.38266416E+01
61	-.4000\000E+01	-.24000000E+02	.30445224E+01	.32580965E+01
62	.1200\000E+02	.23000000E+02	.36109179E+01	.42904594E+01
63	-.1200\000E+02	-.80000000E+01	.25649494E+01	.37376696E+01
64	-.1700\000E+02	.70000000E+01	.20794415E+01	.40430513E+01
65	-.3000\000E+01	-.32000000E+02	.30910425E+01	.28903718E+01
66	-.1200\000E+02	-.26000000E+02	.27080502E+01	.31780538E+01
67	.2000\000E+01	-.37000000E+02	.32958369E+01	.25649494E+01
68	-.9000\000E+01	-.27000000E+02	.29957323E+01	.31354942E+01
69	.3500\000E+02	-.60000000E+01	.40943446E+01	.37376696E+01
73	-.1500\000E+02	.27000000E+02	.23025851E+01	.43438054E+01
71	-.1800\000E+02	-.26000000E+02	.19459101E+01	.31780538E+01
72	-.9900\000E+01	-.37000000E+02	.30910425E+01	.25649494E+01
73	.6000\000E+01	.13000000E+02	.34739872E+01	.41431347E+01
74	.3000\000E+01	-.31000000E+02	.33322045E+01	.29444390E+01
75	-.1500\000E+02	-.27000000E+02	.23025851E+01	.31354942E+01
76	-.1700\000E+02	.50000000E+01	.20794415E+01	.40073332E+01
77	.2000\000E+01	-.15000000E+02	.32958369E+01	.35553481E+01
78	-.9000\000E+01	.80000000E+01	.27725887E+01	.40604430E+01
79	-.1300\000E+02	-.29000000E+02	.24849066E+01	.30445224E+01
83	.5000\000E+01	-.28000000E+02	.34011974E+01	.30910425E+01
81	.1400\000E+02	.20000000E+01	.36635616E+01	.39512437E+01
82	.4000\000E+02	-.37000000E+02	.41743873E+01	.25649494E+01
83	-.1700\000E+02	-.34000000E+02	.20794415E+01	.27725887E+01
84	-.1900\000E+02	-.35000000E+02	.17517595E+01	.27080502E+01
85	-.1100\000E+02	.20000000E+01	.26390573E+01	.39512437E+01
86	-.2000\000E+02	-.30000000E+01	.16094379E+01	.38501476E+01
87	-.5000\000E+01	-.22000000E+02	.29957323E+01	.33322045E+01
88	-.8000\000E+01	.21000000E+02	.28332133E+01	.42626799E+01
89	-.1900\000E+02	-.33000000E+02	.17517595E+01	.28332133E+01
90	-.1800\000E+02	-.15000000E+02	.17517595E+01	.35553481E+01
91	.9100\000E+02	-.37000000E+02	.40253517E+01	.25649494E+01
92	-.1000\000E+02	-.70000000E+01	.27080502E+01	.37612001E+01
93	.1100\000E+02	-.15000000E+02	.35835189E+01	.27080502E+01
94	.1000\000E+02	-.30000000E+02	.35553481E+01	.29957323E+01
95	-.1100\000E+02	-.16000000E+02	.26390573E+01	.35263605E+01
96	-.1700\000E+02	-.37000000E+02	.20794415E+01	.25649494E+01
97	-.1600\000E+02	-.37000000E+02	.21572246E+01	.25649494E+01
98	-.1500\000E+02	-.15000000E+02	.23025851E+01	.35553481E+01
99	-.1200\000E+02	-.24000000E+02	.25649494E+01	.32580965E+01
100	.2000\000E+01	-.30000000E+02	.32958369E+01	.29957323E+01

```
MIN X1 = .16J94279E+01          MAX X1 = .41743873E+01
MIN X2 = .23025851E+01          MAX X2 = .45432948E+01

R=-.15854247E-00   MEAN X1= .28668214E+01   MEAN X2= .32213889E+01   VARIANCE X1= .43577068E-00   VARIANCE X2= .30139972E-00
STAN DEV X1= .66012929E+00      STAN DEV X2= .54899884E+00

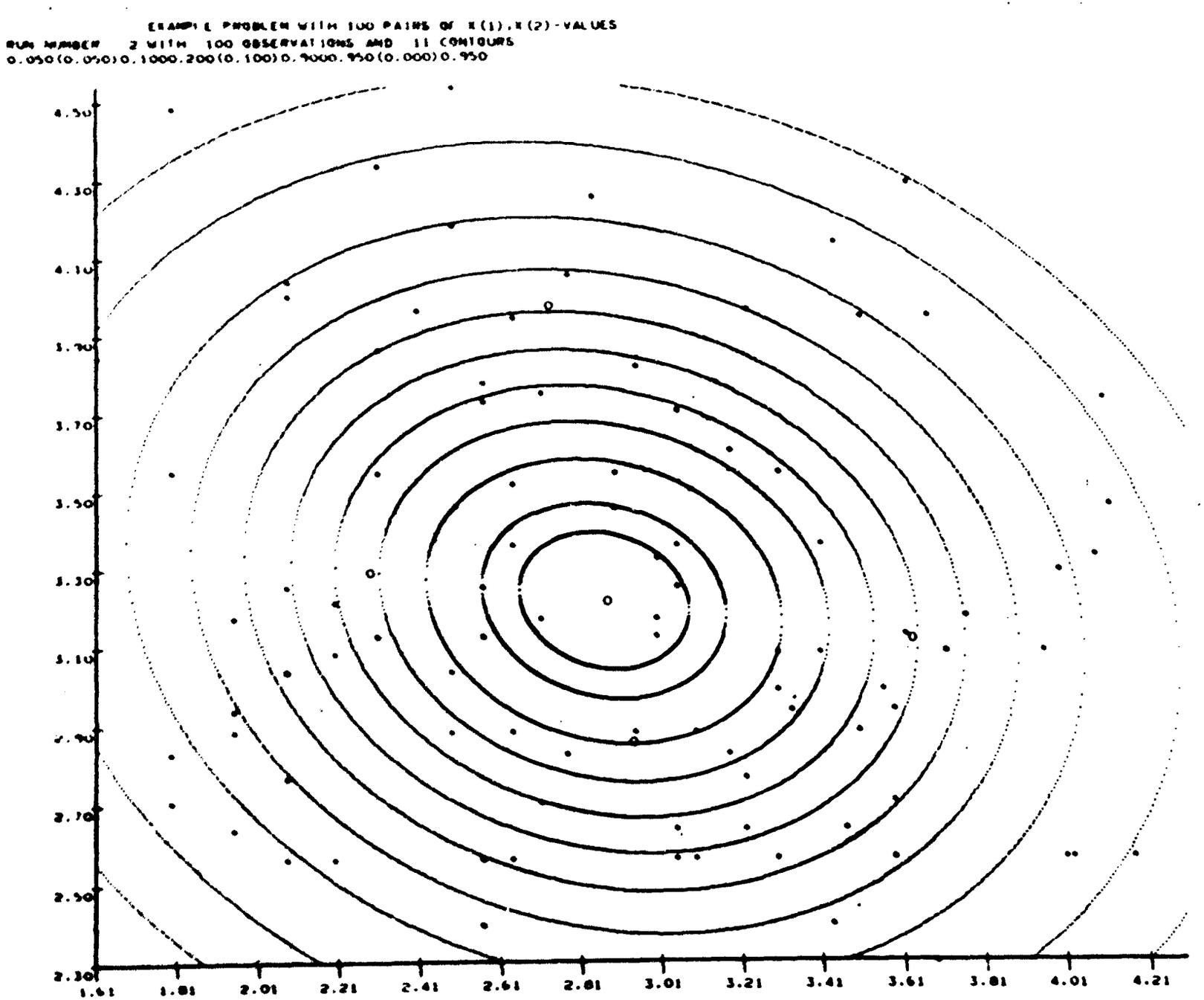
X2= .35993861E+01 + -.13185240E-00 *X1
X1= .34809316E+01 + -.19063524E-00 *X2
```

CHI-SQUARE TEST

RUN NUMBER 2 WITH 100 OBSERVATIONS AND 14 INTERVALS

INTERVAL NUMBER (J)	UPPER BOUND C(J)	PROBABILITY P(J)	OBS FREQ	THEO FREQ	CHI-SQU CONTR
1	.102986599E-00	0.050	5	5.0000	0.0000
2	.210721036E-00	0.100	4	5.0000	0.2000
3	.329337864E-00	0.150	1	5.0000	3.2000
4	.446287106E-00	0.200	5	5.0000	0.0000
5	.713364894E+00	0.300	7	10.0000	0.9000
6	.102165125E+01	0.400	11	10.0000	0.1000
7	.138629436E+01	0.500	11	10.0000	0.1000
8	.181758159E+01	0.600	13	10.0000	0.9000
9	.240794563E+01	0.700	8	10.0000	0.4000
10	.321887582E+01	0.800	15	10.0000	2.5000
11	.479424005E+01	0.850	6	5.0000	0.2000
12	.660917029E+01	0.900	8	5.0000	1.8000
13	.999164459E+01	0.950	5	5.0000	0.0000
14	9.999999999+E16	1.000	1	5.0000	3.2000

CHI-SQUARE = 11.500000 WITH 8 DEGREES OF FREEDOM


EXAMPLE PROBLEM WITH 100 PAIRS OF X(1),X(2)-VALUES
RUN NUMBER 2 WITH 100 OBSERVATIONS AND 11 CONTOURS
0.050(0.050)0.1000.200(0.100)0.9000.950(0.000)0.950
4.50
4.30
4.10
3.90
3.70
3.50
3.30
3.10
2.90
2.70
2.50
2.30
1.61
1.81
2.01
2.21
2.41
2.61
2.81
3.01
3.21
3.41
3.61
3.81
4.01
4.21

```
RANGE OVERFLOW

RANGE OVERFLOW

RANGE OVERFLOW

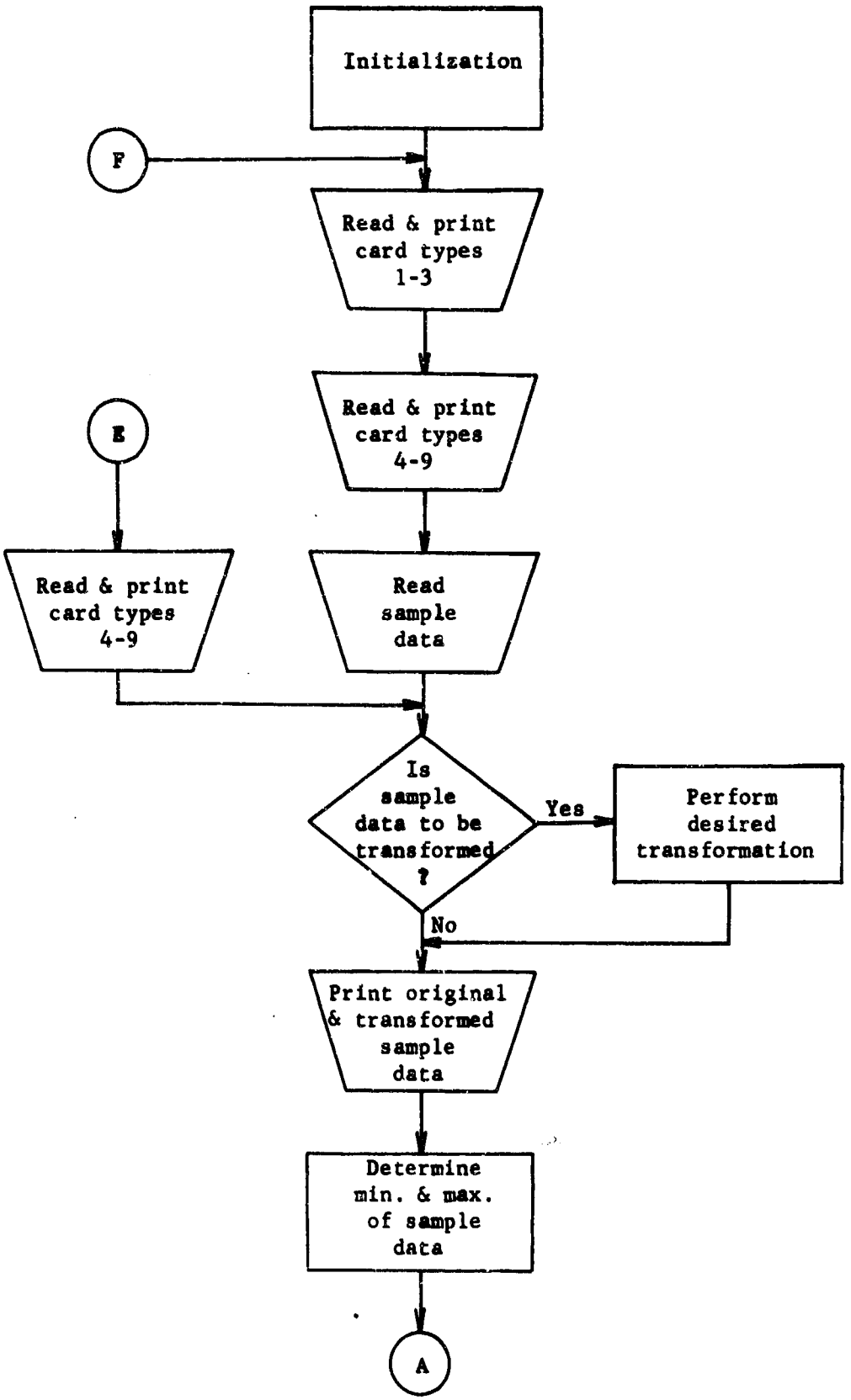
RANGE OVERFLOW
RUNNING TIME IN SECONDS =  .9568898146+01

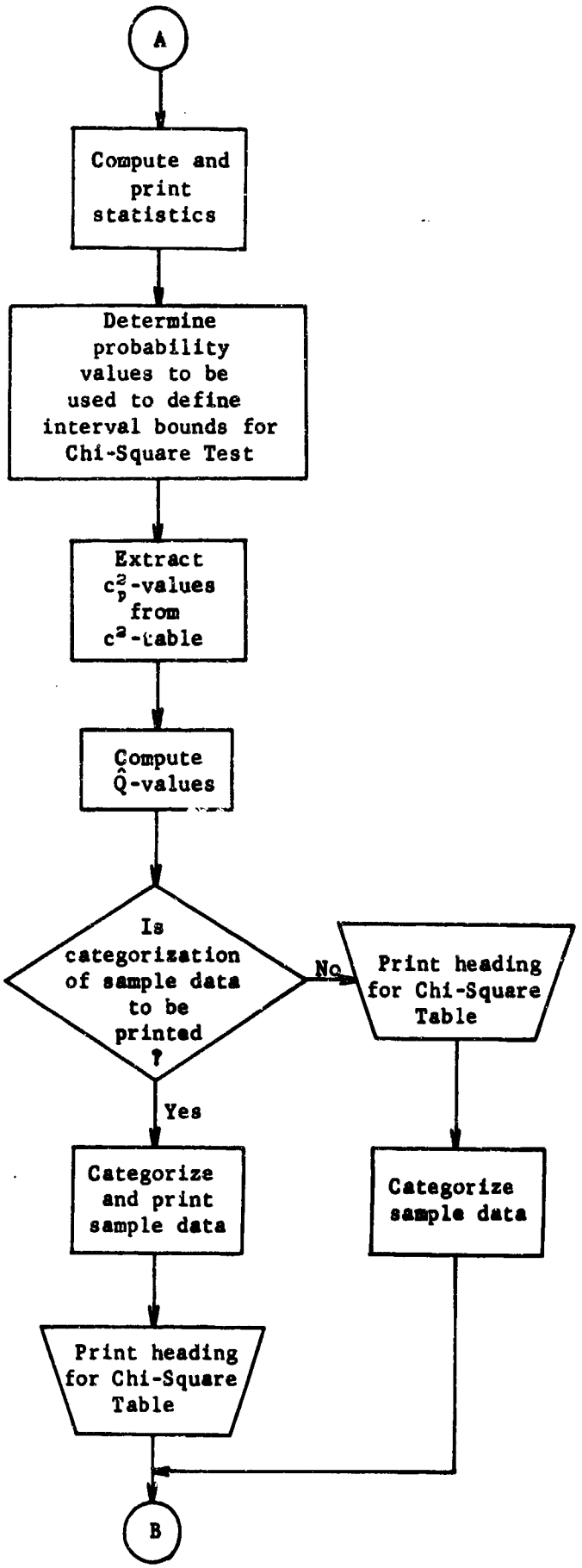

 EOF ON READER
```

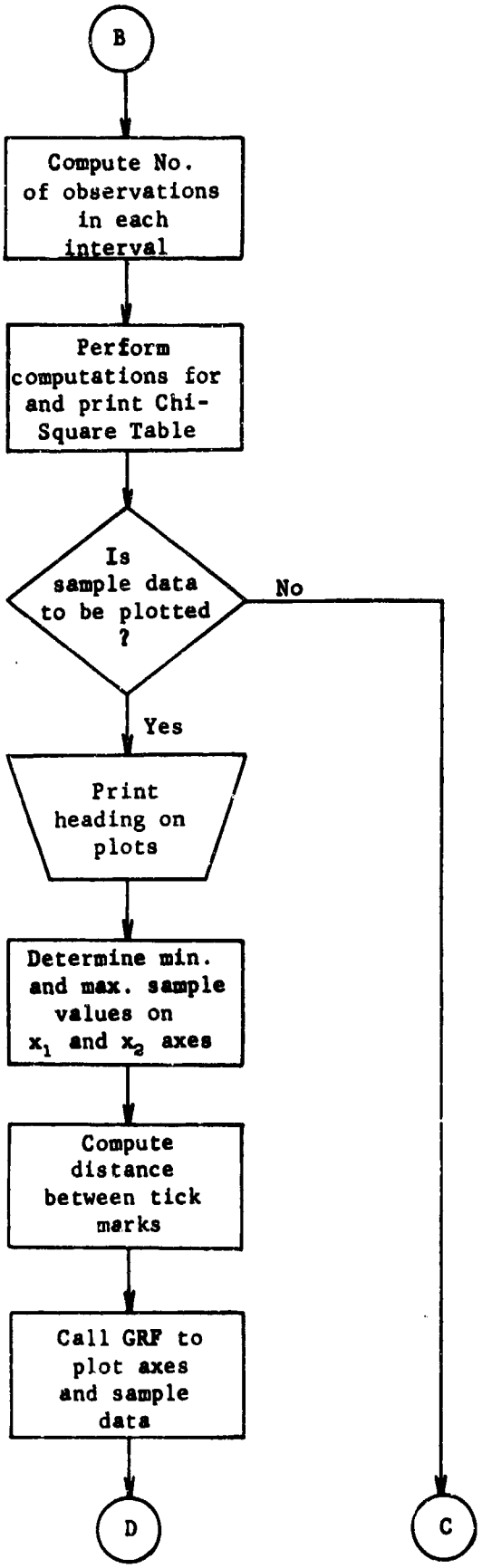
D. Discussion of Test Results

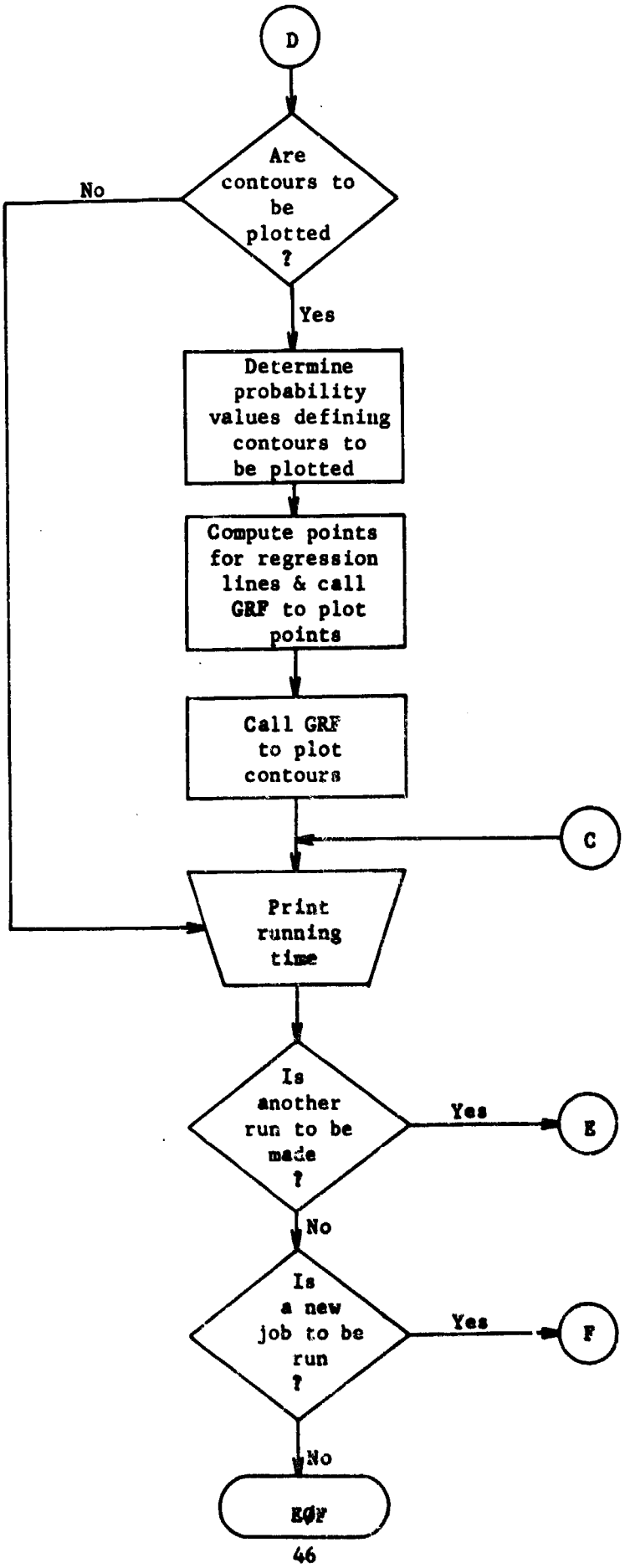
Suppose the 0.05-level of significance is the preselected level for testing the null hypothesis of normality. To test the null hypothesis "The 100 randomly sampled observations are from a parent bivariate normal population," $X^2 = 21.5$ (from the first run) is compared with $\chi^2(0.95,8) = 15.5$. The null hypothesis is, therefore, rejected at the 0.05-level of significance. To test the null hypothesis of normality of the transformed random variables, $X^2 = 13.5$ (from the second run) is compared with $\chi^2(0.95,8) = 15.5$. Because the latter null hypothesis cannot be rejected at the 0.05-level of significance, we conclude that the two original random variables are approximately jointly log normally distributed.

VI. FLOW CHART




A
Compute and print statistics
Determine probability values to be used to define interval bounds for Chi-Square Test
Extract c_p^2-values from c^2-table
Compute $\hat{Q}$-values
Is categorization of sample data to be printed ?
No
Print heading for Chi-Square Table
Yes
Categorize and print sample data
Categorize sample data
Print heading for Chi-Square Table
B


B
Compute No. of observations in each interval
Perform computations for and print Chi-Square Table
Is sample data to be plotted ?
No
Yes
Print heading on plots
Determine min. and max. sample values on x_1 and x_2 axes
Compute distance between tick marks
Call GRF to plot axes and sample data
D
C


D
Are contours to be plotted ?
No
Yes
Determine probability values defining contours to be plotted
Compute points for regression lines & call GRF to plot points
Call GRF to plot contours
C
Print running time
Is another run to be made ?
Yes
E
No
Is a new job to be run ?
Yes
F
No
EOF

VII. PROGRAM LISTING

```
 DIMENSION TABLEC(999),JOB(10),        OBS(999),X1ORIG(4000),        BC000010
1          X2ORIG(4000),PVAL(10),PPVAL(10),TABP(10),TABN(10)        BC000020
 COMMON    PS(10),PDELT(10),PE(10),PPS(10),PPDELT(10),PPE(10),      BC000030
1          X1(4000),X2(4000),P(999),C2(999),Q(4000),HANDY(18),      BC000040
2 FMT(10),XID1(10),XID2(10),IOBS(999),PP(999),E(4000),C2P(999)      BC000050
 DATA (TABP(I),I=1,10)(1.,10.,100.,1000.,10000.,100000.,1000000.,   BC000060
1                      10000000.,100000000.,1000000000.)            BC000070
 DATA(TABN(I),I=1,10)(.000000001,.00000001,.0000001,.000001,        BC000080
1          .00001,.0001,.001,.01,.1,1.)                             BC000090
 DATA (TABLEC(I),I=1,36)                                            BC000100
1(.20010007E-02, .40040053E-02, .60090180E-02, .80160428E-02,       BC000110
2 .10025084E-01, .12036145E-01, .14049230E-01, .16064343E-01,       BC000120
3 .18081489E-01, .20100672E-01, .22121895E-01, .24145162E-01,       BC000130
4 .26170479E-01, .28197849E-01, .30227276E-01, .32258764E-01,       BC000140
5 .34292318E-01, .36327941E-01, .38365639E-01, .40405415E-01,       BC000150
6 .42447273E-01, .44491218E-01, .46537254E-01, .48585385E-01,       BC000160
7 .50635616E-01, .52687951E-01, .54742394E-01, .56798949E-01,       BC000170
8 .58857621E-01, .60918415E-01, .62981334E-01, .65046383E-01,       BC000180
9 .67113567E-01, .69182890E-01, .71254355E-01, .73327969E-01)       BC000190
 DATA (TABLEC(I),I=37,72)                                           BC000200
1(.75403734E-01, .77481657E-01, .79561740E-01, .81643989E-01,       BC000210
2 .83728408E-01, .85815002E-01, .87903775E-01, .89994732E-01,       BC000220
3 .92087877E-01, .94183215E-01, .96280751E-01, .98380488E-01,       BC000230
4 .10048243E-00, .10258659E-00, .10469296E-00, .10680155E-00,       BC000240
5 .10891237E-00, .11102542E-00, .11314070E-00, .11525823E-00,       BC000250
6 .11737799E-00, .11950001E-00, .12162428E-00, .12375081E-00,       BC000260
7 .12587960E-00, .12801066E-00, .13014399E-00, .13227961E-00,       BC000270
8 .13441750E-00, .13655768E-00, .13870016E-00, .14084493E-00,       BC000280
9 .14299200E-00, .14514139E-00, .14729308E-00, .14944709E-00)       BC000290
 DATA (TABLEC(I),I=73,108)                                          BC000300
1(.15160343E-00, .15376209E-00, .15592308E-00, .15808641E-00,       BC000310
2 .16025209E-00, .16242011E-00, .16459049E-00, .16676322E-00,       BC000320
3 .16893831E-00, .17111578E-00, .17329561E-00, .17547783E-00,       BC000330
4 .17766243E-00, .17984942E-00, .18203880E-00, .18423058E-00,       BC000340
5 .18642476E-00, .18862136E-00, .19082037E-00, .19302180E-00,       BC000350
6 .19522566E-00, .19743195E-00, .19964067E-00, .20185184E-00,       BC000360
7 .20406545E-00, .20628152E-00, .20850004E-00, .21072103E-00,       BC000370
8 .21294449E-00, .21517042E-00, .21739883E-00, .21962973E-00,       BC000380
9 .22186312E-00, .22409901E-00, .22633740E-00, .22857829E-00)       BC000390
 DATA (TABLEC(I),I=109,144)                                         BC000400
1(.23082170E-00, .23306763E-00, .23531609E-00, .23756707E-00,       BC000410
2 .23982059E-00, .24207666E-00, .24433527E-00, .24659643E-00,       BC000420
3 .24886016E-00, .25112645E-00, .25339531E-00, .25566674E-00,       BC000430
4 .25794076E-00, .26021737E-00, .26249657E-00, .26477838E-00,       BC000440
5 .26706279E-00, .26934981E-00, .27163945E-00, .27393171E-00,       BC000450
6 .27622660E-00, .27852413E-00, .28082431E-00, .28312713E-00,       BC000460
7 .28543260E-00, .28774074E-00, .29005154E-00, .29236502E-00,       BC000470
8 .29468118E-00, .29700002E-00, .29932155E-00, .30164578E-00,       BC000480
9 .30397271E-00, .30630236E-00, .30863472E-00, .31096981E-00)       BC000490
 DATA (TABLEC(I),I=145,180)                                         BC000500
1(.31330762E-00, .31564817E-00, .31799146E-00, .32033750E-00,       BC000510
2 .32268630E-00, .32503786E-00, .32739219E-00, .32974929E-00,       BC000520
3 .33210917E-00, .33447184E-00, .33683730E-00, .33920557E-00,       BC000530
4 .34157664E-00, .34395053E-00, .34632724E-00, .34870677E-00,       BC000540
5 .35108915E-00, .35347436E-00, .35586242E-00, .35825333E-00,       BC000550
6 .36064711E-00, .36304375E-00, .36544327E-00, .36784568E-00,       BC000560
7 .37025097E-00, .37265916E-00, .37507025E-00, .37748425E-00,       BC000570
8 .37990117E-00, .38232101E-00, .38474379E-00, .38716950E-00,       BC000580
9 .38959816E-00, .39202977E-00, .39446434E-00, .39690188E-00)       BC000590
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      DATA (TABLEC(I),I=181,216)                                        BC000600
     1(.39934239E-00, .40178588E-00, .40423237E-00, .40668185E-00,      BC000610
     2 .40913433E-00, .41158983E-00, .41404834E-00, .41650988E-00,      BC000620
     3 .41897445E-00, .42144206E-00, .42391272E-00, .42638644E-00,      BC000630
     4 .42886322E-00, .43134307E-00, .43382600E-00, .43631202E-00,      BC000640
     5 .43880113E-00, .44129334E-00, .44378866E-00, .44628710E-00,      BC000650
     6 .44878867E-00, .45129336E-00, .45380120E-00, .45631219E-00,      BC000660
     7 .45882633E-00, .46134364E-00, .46386411E-00, .46638777E-00,      BC000670
     8 .46891462E-00, .47144467E-00, .47397792E-00, .47651438E-00,      BC000680
     9 .47905406E-00, .48159697E-00, .48414312E-00, .48669252E-00)      BC000690
      DATA (TABLEC(I),I=217,252)                                        BC000700
     1(.48924517E-00, .49180108E-00, .49436026E-00, .49692272E-00,      BC000710
     2 .49948847E-00, .50205751E+00, .50462986E+00, .50720552E+00,      BC000720
     3 .50978450E+00, .51236681E+00, .51495246E+00, .51754146E+00,      BC000730
     4 .52013381E+00, .52272953E+00, .52532862E+00, .52793109E+00,      BC000740
     5 .53053696E+00, .53314622E+00, .53575889E+00, .53837498E+00,      BC000750
     6 .54099450E+00, .54361745E+00, .54624384E+00, .54887369E+00,      BC000760
     7 .55150700E+00, .55414379E+00, .55678405E+00, .55942781E+00,      BC000770
     8 .56207506E+00, .56472582E+00, .56738010E+00, .57003791E+00,      BC000780
     9 .57269925E+00, .57536414E+00, .57803259E+00, .58070460E+00)      BC000790
      DATA (TABLEC(I),I=253,288)                                        BC000800
     1 .58338019E+00, .58605936E+00, .58874212E+00, .59142849E+00,      BC000810
     2 .59411847E+00, .59681207E+00, .59950931E+00, .60221019E+00,      BC000820
     3 .60491472E+00, .60762291E+00, .61033477E+00, .61305032E+00,      BC000830
     4 .61576956E+00, .61849250E+00, .62121915E+00, .62394953E+00,      BC000840
     5 .62668364E+00, .62942149E+00, .63216309E+00, .63490846E+00,      BC000850
     6 .63765760E+00, .64041053E+00, .64316725E+00, .64592777E+00,      BC000860
     7 .64869211E+00, .65146028E+00, .65423228E+00, .65700813E+00,      BC000870
     8 .65978784E+00, .66257142E+00, .66535888E+00, .66815022E+00,      BC000880
     9 .67094547E+00, .67374463E+00, .67654772E+00, .67935474E+00)      BC000890
      DATA (TABLEC(I),I=289,324)                                        BC000900
     1(.68216570E+00, .68498062E+00, .68779950E+00, .69062237E+00,      BC000910
     2 .69344923E+00, .69628008E+00, .69911495E+00, .70195385E+00,      BC000920
     3 .70479677E+00, .70764375E+00, .71049478E+00, .71334989E+00,      BC000930
     4 .71620907E+00, .71907235E+00, .72193974E+00, .72481124E+00,      BC000940
     5 .72768687E+00, .73056664E+00, .73345056E+00, .73633865E+00,      BC000950
     6 .73923091E+00, .74212736E+00, .74502802E+00, .74793288E+00,      BC000960
     7 .75084197E+00, .75375530E+00, .75667288E+00, .75959472E+00,      BC000970
     8 .76252084E+00, .76545124E+00, .76838595E+00, .77132496E+00,      BC000980
     9 .77426830E+00, .77721598E+00, .78016801E+00, .78312441E+00)      BC000990
      DATA (TABLEC(I),I=325,360)                                        BC001000
     1(.78608518E+00, .78905034E+00, .79201990E+00, .79499388E+00,      BC001010
     2 .79797228E+00, .80095513E+00, .80394244E+00, .80693421E+00,      BC001020
     3 .80993047E+00, .81293122E+00, .81593648E+00, .81894626E+00,      BC001030
     4 .82196058E+00, .82497945E+00, .82800288E+00, .83103089E+00,      BC001040
     5 .83406349E+00, .83710070E+00, .84014252E+00, .84318898E+00,      BC001050
     6 .84624009E+00, .84929586E+00, .85235630E+00, .85542143E+00,      BC001060
     7 .85849127E+00, .86156583E+00, .86464512E+00, .86772917E+00,      BC001070
     8 .87081797E+00, .87391155E+00, .87700992E+00, .88011311E+00,      BC001080
     9 .88322111E+00, .88633395E+00, .88945164E+00, .89257421E+00)      BC001090
      DATA (TABLEC(I),I=361,396)                                        BC001100
     1(.89570165E+00, .89883399E+00, .90197125E+00, .90511343E+00,      BC001110
     2 .90826056E+00, .91141265E+00, .91456971E+00, .91773177E+00,      BC001120
     3 .92089883E+00, .92407092E+00, .92724804E+00, .93043023E+00,      BC001130
     4 .93361748E+00, .93680982E+00, .94000726E+00, .94320982E+00,      BC001140
     5 .94641752E+00, .94963037E+00, .95284839E+00, .95607160E+00,      BC001150
     6 .95930001E+00, .96253364E+00, .96577251E+00, .96901663E+00,      BC001160
     7 .97226602E+00, .97552070E+00, .97878069E+00, .98204599E+00,      BC001170
     8 .98531664E+00, .98859264E+00, .99187402E+00, .99516079E+00,      BC001180
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9 .99845298E+00, .10017506E+01, .10050536E+01, .10083624E+01)      BC001190
 DATA (TABLEC(I),I=397,432)                                        BC001200
1(.10116762E+01, .10149957E+01, .10183207E+01, .10216512E+01,      BC001210
2 .10249874E+01, .10283291E+01, .10316763E+01, .10350292E+01,      BC001220
3 .10383877E+01, .10417519E+01, .10451218E+01, .10484973E+01,      BC001230
4 .10518785E+01, .10552655E+01, .10586582E+01, .10620567E+01,      BC001240
5 .10654609E+01, .10688710E+01, .10722869E+01, .10757086E+01,      BC001250
6 .10791362E+01, .10825697E+01, .10860090E+01, .10894544E+01,      BC001260
7 .10929056E+01, .10963628E+01, .10998260E+01, .11032952E+01,      BC001270
8 .11067705E+01, .11102518E+01, .11137391E+01, .11172326E+01,      BC001280
9 .11207321E+01, .11242378E+01, .11277497E+01, .11312677E+01)      BC001290
 DATA (TABLEC(I),I=433,468)                                        BC001300
1(.11347920E+01, .11383224E+01, .11418591E+01, .11454021E+01,      BC001310
2 .11489513E+01, .11525069E+01, .11560687E+01, .11596370E+01,      BC001320
3 .11632116E+01, .11667926E+01, .11703801E+01, .11739740E+01,      BC001330
4 .11775743E+01, .11811812E+01, .11847946E+01, .11884145E+01,      BC001340
5 .11920409E+01, .11956740E+01, .11993137E+01, .12029600E+01,      BC001350
6 .12066130E+01, .12102726E+01, .12139390E+01, .12176121E+01,      BC001360
7 .12212919E+01, .12249786E+01, .12286720E+01, .12323723E+01,      BC001370
8 .12360794E+01, .12397934E+01, .12435144E+01, .12472422E+01,      BC001380
9 .12509771E+01, .12547189E+01, .12584677E+01, .12622236E+01)      BC001390
 DATA (TABLEC(I),I=469,504                                         BC001400
1(.12659865E+01, .12697565E+01, .12735337E+01, .12773180E+01,      BC001410
2 .12811095E+01, .12849081E+01, .12887140E+01, .12925272E+01,      BC001420
3 .12963476E+01, .13001754E+01, .13040105E+01, .13078529E+01,      BC001430
4 .13117028E+01, .13155601E+01, .13194248E+01, .13232970E+01,      BC001440
5 .13271768E+01, .13310640E+01, .13349589E+01, .13388613E+01,      BC001450
6 .13427714E+01, .13466891E+01, .13506145E+01, .13545477E+01,      BC001460
7 .13584886E+01, .13624372E+01, .13663937E+01, .13703580E+01,      BC001470
8 .13743302E+01, .13783103E+01, .13822984E+01, .13862944E+01,      BC001480
9 .13902984E+01, .13943104E+01, .13983305E+01, .14023587E+01)      BC001490
 DATA (TABLEC(I),I=505,540)                                        BC001500
1(.14063950E+01, .14104395E+01, .14144922E+01, .14185532E+01,      BC001510
2 .14226223E+01, .14266998E+01, .14307856E+01, .14348797E+01,      BC001520
3 .14389823E+01, .14430933E+01, .14472128E+01, .14513407E+01,      BC001530
4 .14554773E+01, .14596223E+01, .14637760E+01, .14679384E+01,      BC001540
5 .14721094E+01, .14762891E+01, .14804776E+01, .14846748E+01,      BC001550
6 .14888809E+01, .14930959E+01, .14973198E+01, .15015526E+01,      BC001560
7 .15057944E+01, .15100452E+01, .15143050E+01, .15185740E+01,      BC001570
8 .15228520E+01, .15271393E+01, .15314357E+01, .15357415E+01,      BC001580
9 .15400564E+01, .15443808E+01, .15487145E+01, .15530576E+01)      BC001590
 DATA (TABLEC(I),I=541,576)                                        BC001600
1(.15574101E+01, .15617722E+01, .15661438E+01, .15705249E+01,      BC001610
2 .15749157E+01, .15793162E+01, .15837263E+01, .15881462E+01,      BC001620
3 .15925759E+01, .15970154E+01, .16014648E+01, .16059241E+01,      BC001630
4 .16103934E+01, .16148727E+01, .16193620E+01, .16238614E+01,      BC001640
5 .16283710E+01, .16328908E+01, .16374208E+01, .16419611E+01,      BC001650
6 .16465117E+01, .16510727E+01, .16556442E+01, .16602261E+01,      BC001660
7 .16648185E+01, .16694215E+01, .16740351E+01, .16786594E+01,      BC001670
8 .16832944E+01, .16879401E+01, .16925967E+01, .16972642E+01,      BC001680
9 .17019425E+01, .17066319E+01, .17113322E+01, .17160436E+01)      BC001690
 DATA (TABLEC(I),I=577,612)                                        BC001700
1(.17207662E+01, .17254999E+01, .17302449E+01, .17350011E+01,      BC001710
2 .17397687E+01, .17445477E+01, .17493381E+01, .17541400E+01,      BC001720
3 .17589535E+01, .17637786E+01, .17686154E+01, .17734639E+01,      BC001730
4 .17783241E+01, .17831962E+01, .17880802E+01, .17929762E+01,      BC001740
5 .17978842E+01, .18028042E+01, .18077364E+01, .18126808E+01,      BC001750
6 .18176374E+01, .18226064E+01, .18275877E+01, .18325815E+01,      BC001760
7 .18375877E+01, .18426065E+01, .18476380E+01, .18526821E+01,      BC001770
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8 .18[illegible]7390E+01, .18628087E+01, .18678913E+01, .18729869E+01,      BC001780
9 .18780954E+01, .18832171E+01, .18883519E+01, .18934999E+01)              BC001790
 DATA (TABLEC(I),I=613,648)                                               BC001800
1(.18986612E+01, .19038358E+01, .19090239E+01, .19142255E+01,             BC001810
2 .19194406E+01, .19246693E+01, .19299118E+01, .19351681E+01,             BC001820
3 .19404381E+01, .19457222E+01, .19510202E+01, .19563323E+01,             BC001830
4 .19616585E+01, .19669990E+01, .19723537E+01, .19777228E+01,             BC001840
5 .19831064E+01, .19885045E+01, .19939173E+01, .19993447E+01,             BC001850
6 .20047869E+01, .20102439E+01, .20157159E+01, .20212028E+01,             BC001860
7 .20267049E+01, .20322221E+01, .20377546E+01, .20433025E+01,             BC001870
8 .20488658E+01, .20544446E+01, .20600390E+01, .20656491E+01,             BC001880
9 .20712750E+01, .20769167E+01, .20825744E+01, .20882482E+01)             BC001890
 DATA (TABLEC(I),I=649,684)                                               BC001900
1(.20939381E+01, .20996442E+01, .21053667E+01, .21111056E+01,             BC001910
2 .21168610E+01, .21226330E+01, .21284217E+01, .21342272E+01,             BC001920
3 .21400497E+01, .21458891E+01, .21517456E+01, .21576193E+01,             BC001930
4 .21635103E+01, .21694188E+01, .21753447E+01, .21812882E+01,             BC001940
5 .21872495E+01, .21932286E+01, .21992256E+01, .22052406E+01,             BC001950
6 .22112738E+01, .22173252E+01, .22233951E+01, .22294833E+01,             BC001960
7 .22355902E+01, .22417158E+01, .22478602E+01, .22540235E+01,             BC001970
8 .22602059E+01, .22664075E+01, .22726283E+01, .22788686E+01,             BC001980
9 .22851284E+01, .22914078E+01, .22977070E+01, .23040261E+01)             BC001990
 DATA (TABLEC(I),I=685,720)                                               BC002000
1(.23103653E+01, .23167246E+01, .23231042E+01, .23295042E+01,             BC002010
2 .23359247E+01, .23423660E+01, .23488280E+01, .23553110E+01,             BC002020
3 .23618151E+01, .23683404E+01, .23748870E+01, .23814552E+01,             BC002030
4 .23880449E+01, .23946565E+01, .24012900E+01, .24079456E+01,             BC002040
5 .24146234E+01, .24213236E+01, .24280463E+01, .24347916E+01,             BC002050
6 .24415598E+01, .24483510E+01, .24551653E+01, .24620030E+01,             BC002060
7 .24688640E+01, .24757487E+01, .24826572E+01, .24895896E+01,             BC002070
8 .24965461E+01, .25035269E+01, .25105322E+01, .25175621E+01,             BC002080
9 .25246168E+01, .25316964E+01, .25388012E+01, .25459314E+01)             BC002090
 DATA (TABLEC(I),I=721,756)                                               BC002100
1(.25530870E+01, .25602683E+01, .25674755E+01, .25747088E+01,             BC002110
2 .25819684E+01, .25892543E+01, .25965670E+01, .26039064E+01,             BC002120
3 .26112729E+01, .26186666E+01, .26260878E+01, .26335366E+01,             BC002130
4 .26410132E+01, .26485179E+01, .26560509E+01, .26636124E+01,             BC002140
5 .26712025E+01, .26788216E+01, .26864697E+01, .26941473E+01,             BC002150
6 .27018544E+01, .27095914E+01, .27173584E+01, .27251557E+01,             BC002160
7 .27329835E+01, .27408420E+01, .27487316E+01, .27566524E+01,             BC002170
8 .27646047E+01, .27725887E+01, .27806048E+01, .27886531E+01,             BC002180
9 .27967339E+01, .28048475E+01, .28129941E+01, .28211741E+01)             BC002190
 DATA (TABLEC(I),I=757,792)                                               BC002200
1(.28293877E+01, .28376351E+01, .28459167E+01, .28542327E+01,             BC002210
2 .28625835E+01, .28709692E+01, .28793903E+01, .28878469E+01,             BC002220
3 .28963395E+01, .29048683E+01, .29134337E+01, .29220358E+01,             BC002230
4 .29306751E+01, .29393519E+01, .29480666E+01, .29568193E+01,             BC002240
5 .29656105E+01, .29744406E+01, .29833098E+01, .29922185E+01,             BC002250
6 .30011670E+01, .30101558E+01, .30191852E+01, .30282555E+01,             BC002260
7 .30373671E+01, .30465204E+01, .30557159E+01, .30649537E+01,             BC002270
8 .30742345E+01, .30835585E+01, .30929262E+01, .31023380E+01,             BC002280
9 .31117943E+01, .31212955E+01, .31308421E+01, .31404344E+01)             BC002290
 DATA (TABLEC(I),I=793,828)                                               BC002300
1(.31500730E+01, .31597582E+01, .31694906E+01, .31792706E+01,             BC002310
2 .31890986E+01, .31989752E+01, .32089007E+01, .32188758E+01,             BC002320
3 .32289009E+01, .32389765E+01, .32491031E+01, .32592812E+01,             BC002330
4 .32695114E+01, .32797942E+01, .32901302E+01, .33005198E+01,             BC002340
5 .33109637E+01, .33214624E+01, .33320165E+01, .33426266E+01,             BC002350
6 .33532933E+01, .33640172E+01, .33747989E+01, .33856390E+01,             BC002360
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     7 .33965383E+01, .34074972E+01, .34185165E+01, .34295969E+01,      BC002370
     8 .34407389E+01, .34519435E+01, .34632111E+01, .34745426E+01,      BC002380
     9 .34859386E+01, .34974000E+01, .35089274E+01, .35205216E+01)      BC002390
      DATA (TABLEC(I),I=829,864)                                        BC002400
     1(.35321834E+01, .35439137E+01, .35557131E+01, .35675826E+01,      BC002410
     2 .35795229E+01, .35915350E+01, .36036196E+01, .36157777E+01,      BC002420
     3 .36280102E+01, .36403179E+01, .36527018E+01, .36651629E+01,      BC002430
     4 .36777022E+01, .36903205E+01, .37030189E+01, .37157985E+01,      BC002440
     5 .37286603E+01, .37416054E+01, .37546347E+01, .37677495E+01,      BC002450
     6 .37809509E+01, .37942400E+01, .38076179E+01, .38210860E+01,      BC002460
     7 .38346454E+01, .38482973E+01, .38620431E+01, .38758840E+01,      BC002470
     8 .38898213E+01, .39038564E+01, .39179908E+01, .39322257E+01,      BC002480
     9 .39465627E+01, .39610032E+01, .39755487E+01, .39902008E+01)      BC002490
      DATA (TABLEC(I),I=865,900)                                        BC002500
     1(.40049610E+01, .40198310E+01, .40348123E+01, .40499067E+01,      BC002510
     2 .40651159E+01, .40804417E+01, .40958857E+01, .41114500E+01,      BC002520
     3 .41271364E+01, .41429467E+01, .41588831E+01, .41749474E+01,      BC002530
     4 .41911418E+01, .42074685E+01, .42239295E+01, .42405271E+01,      BC002540
     5 .42572636E+01, .42741413E+01, .42911627E+01, .43083302E+01,      BC002550
     6 .43256463E+01, .43431137E+01, .43607349E+01, .43785128E+01,      BC002560
     7 .43964502E+01, .44145498E+01, .44328148E+01, .44512481E+01,      BC002570
     8 .44698529E+01, .44886324E+01, .45075899E+01, .45267288E+01,      BC002580
     9 .45460526E+01, .45655649E+01, .45852695E+01, .46051702E+01)      BC002590
      DATA (TABLEC(I),I=901,936)                                        BC002600
     1(.46252709E+01, .46455756E+01, .46660886E+01, .46868142E+01,      BC002610
     2 .47077568E+01, .47289210E+01, .47503116E+01, .47719334E+01,      BC002620
     3 .47937915E+01, .48158912E+01, .48382378E+01, .48608369E+01,      BC002630
     4 .48836943E+01, .49068160E+01, .49302080E+01, .49538770E+01,      BC002640
     5 .49778293E+01, .50020721E+01, .50266122E+01, .50514573E+01,      BC002650
     6 .50766149E+01, .51020929E+01, .51278997E+01, .51540439E+01,      BC002660
     7 .51805343E+01, .52073804E+01, .52345917E+01, .52621783E+01,      BC002670
     8 .52901508E+01, .53185201E+01, .53472975E+01, .53764951E+01,      BC002680
     9 .54061253E+01, .54362011E+01, .54667360E+01, .54977444E+01)      BC002690
      DATA (TABLEC(I),I=937,972)                                        BC002700
     1(.55292411E+01, .55612418E+01, .55937628E+01, .56268214E+01,      BC002710
     2 .56604357E+01, .56946243E+01, .57294080E+01, .57648072E+01,      BC002720
     3 .58008442E+01, .58375425E+01, .58749267E+01, .59130231E+01,      BC002730
     4 .59518593E+01, .59914645E+01, .60318700E+01, .60731085E+01,      BC002740
     5 .61152154E+01, .61582278E+01, .62021856E+01, .62471313E+01,      BC002750
     6 .62931103E+01, .63401713E+01, .63883664E+01, .64377516E+01,      BC002760
     7 .64883873E+01, .65403382E+01, .65936747E+01, .66484727E+01,      BC002770
     8 .67048144E+01, .67627895E+01, .68224954E+01, .68840388E+01,      BC002780
     9 .69475361E+01, .70131150E+01, .70809189E+01, .71511015E+01)      BC002790
      DATA (TABLEC(I),I=973,999)                                        BC002800
     1(.72238769E+01, .72993175E+01, .73777589E+01, .74594029E+01,      BC002810
     2 .75445221E+01, .76334257E+01, .77264557E+01, .78240460E+01,      BC002820
     3 .79266326E+01, .80347670E+01, .81490839E+01, .82703331E+01,      BC002830
     4 .83994025E+01, .85373289E+01, .86856118E+01, .88456973E+01,      BC002840
     5 .90197205E+01, .92103404E+01, .94210614E+01, .96566275E+01,      BC002850
     6 .99236923E+01, .10231925E+02, .10596635E+02, .11042922E+02,      BC002860
     7 .11618286E+02, .12429216E+02, .13815511E+02)                     BC002870
      EQUIVALENCE (OBS,IGAS)                                            BC002980
      CALL SETIT                                                        BC002890
      CALL CRTID (4HA38C)                                               BC002900
      C506=5./6.                                                        BC002910
      C605=6./5.                                                        BC002920
      LA=1                                                              BC002930
      LO=1                                                              BC002940
   50 READ 100,JOB,FMT                                                  BC002950
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1[illegible] FORMAT (10A8/10A8)                                                  BC002960
      READ 115,NRUN                                                              BC002970
115   FORMAT (I5)                                                                BC002980
      DO 12[illegible] IRUN=1,NRUN                                               BC002990
      PRINT 11[illegible],JOB,FMT                                                BC003000
110   FORMAT (1H1,56X,13HCONTROL CARDS//10A8//2X,20HVARIABLE FORMAT CARDBC003010
     1,5X,10A8)                                                                  BC003020
      PRINT 117,NRUN                                                             BC003030
117   FORMAT (1H0, 6HNRUN =,I3)                                                  BC003040
120   READ 150,NPRINT,JPLOT,NOTIC,INPUT,NTYPES,ITRAN1,ITRAN2,IRA,IRO,            BC003050
     1          NPAIR,NN,NL,(PS(JJ),PDELT(JJ),PE(JJ),JJ=1,NN)                   BC003060
150   FORMAT (12I5/15F5.3)                                                       BC003070
      PRINT 155,IRUN,NPRINT,JPLOT,NOTIC,INPUT,NTYPES,ITRAN1,ITRAN2,IRA,          BC003080
     1          IRO,NPAIR,NN,NL                                                 BC003090
155   FORMAT (1H0,6HIRUN =,I3/1H0,9HNPRINT = ,I1,5X,8HJPLOT = ,I1,5X,            BC003100
     1       8HNOTIC = ,I2,5X,8HINPUT = ,I1,5X,8HNTYPES = ,I1,5X,               BC003110
     2       9HITRAN1 = ,I2/1H0,9HITRAN2 = ,I2,4X,6HIRA = ,I1,7X,               BC003120
     3       6HIRO = ,I1, 8X,8HNPAIR = ,I2,4X,5HNN = ,I1,9X,5HNL = ,I1)         BC003130
      PRINT 157,(PS(JJ),PDELT(JJ),PE(JJ),JJ=1,NN)                                BC003140
157   FORMAT (1H0,43HP-VALUES IDENTIFICATION FOR CHI-SQUARE TEST/1H ,5X,         BC003150
     1        5(F5.3,1H(,F5.3,1H),F5.3,2X))                                     BC003160
      IF (JPLOT.LE.0) GO TO 163                                                  BC003170
      IF (JPLOT.EQ.2) GO TO 163                                                  BC003180
      READ 160, (PPS(JJ),PPDELT(JJ),PPE(JJ),JJ=1,NL)                             BC003190
160   FORMAT ((15F5.3))                                                          BC003200
      PRINT 162,(PPS(JJ),PPDELT(JJ),PPE(JJ),JJ=1,NL)                             BC003210
162   FORMAT (1H0,33HP-VALUES IDENTIFICATION FOR PLOTS/1H ,5X,5(F5.3,             BC003220
     1       1H(,F5.3,1H),F5.3,2X ))                                            BC003230
163   IF (NTYPES.LE.0) GO TO 165                                                 BC003240
      READ 164, X1D1,X1D2                                                        BC003250
164   FORMAT (10A8/10A8)                                                         BC003260
165   IF (ITRAN1.EQ.1) GO TO 168                                                 BC003270
      READ 166,ATRAN1,BTRAN1,CTRAN1,DTRAN1,ETRAN1                                BC003280
166   FORMAT (5E14.8)                                                            BC003290
      PRINT 167,ITRAN1,ATRAN1,BTRAN1,CTRAN1,DTRAN1,ETRAN1                        BC003300
167   FORMAT (1H0,37HTRANSFORMATION CODE FOR X1 VALUES IS ,I2/1H ,                BC003310
     1        43HTRANSFORMATION CONSTANTS FOR X1 VALUES ARE-/1H ,3X,             BC003320
     2        8HATRAN1 =,E15.8,3X,8HBTRAN1 =,E15.8,3X,8HCTRAN1 =,E15.8           BC003330
     3        ,3X,8HDTRAN1 =,E15.8,3X,8HETRAN1 =,E15.8)                          BC003340
168   IF(ITRAN2.EQ.1) GO TO 180                                                  BC003350
      READ 169,ATRAN2,BTRAN2,CTRAN2,DTRAN2,ETRAN2                                BC003360
169   FORMAT (5E14.8)                                                            BC003370
      PRINT 170,ITRAN2,ATRAN2,BTRAN2,CTRAN2,DTRAN2,ETRAN2                        BC003380
170   FORMAT (1H0,37HTRANSFORMATION CODE FOR X2 VALUES IS ,I2/1H ,                BC003390
     1        43HTRANSFORMATION CONSTANTS FOR X2 VALUES ARE-/1H ,3X,             BC003400
     2        8HATRAN2 =,E15.8,3X,8HBTRAN2 =,E15.8,3X,8HCTRAN2 =,E15.8,          BC003410
     3        3X,8HDTRAN2 =,E15.8,3X,8HETRAN2 =,E15.8)                           BC003420
180   PRINT 190                                                                  BC003430
190   FORMAT (1H1,15X,13HORIGINAL DATA,19X,16HTRANSFORMED DATA/1H ,3X,           BC003440
     1       1HI,8X,2HX1,14X,2HX2,15X,2HX1,14X,2HX2)                            BC003450
      IF(IRUN.GT.1) GO TO 701                                                    BC003460
200   I1=1                                                                       BC003470
      I2=NPAIR                                                                   BC003480
      IF (INPUT.LE.0) GO TO 500                                                  BC003490
300   READ (5,FMT) IEND,(X1(I),X2(I),I=I1,I2)                                   BC003500
      IF (IEND.NE.0) GO TO 400                                                   BC003510
      I1=I1+NPAIR                                                                BC003520
      I2=I2+NPAIR                                                                BC003530
      GO TO 300                                                                  BC003540
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400 NPTS=I1+IEND-1                                                      BC003550
    GO TO 700                                                           BC003560
500 READ FMT,IEND,(X1(I),X2(I),I=I1,I2)                                 BC003570
    IF (IEND.NE.0) GO TO 600                                            BC003580
    I1=I1+NPAIR                                                         BC003590
    I2=I2+NPAIR                                                         BC003600
    GO TO 500                                                           BC003610
600 NPTS =I1+IEND-1                                                     BC003620
700 CALL SEND (NPTS,X1,X1ORIG)                                          BC003630
    CALL SEND (NPTS,X2,X2ORIG)                                          BC003640
701 IF (ITRAN1.LE.1) GO TO 825                                          BC003650
    GO TO (825,705,715,725,735,745,755,765,775,785,795,805,815),ITRAN1BC003660
705 DO 710  K=1,NPTS                                                    BC003670
710 X1(K)=ALOG(X1(K))                                                   BC003680
    GO TO 825                                                           BC003690
715 DO 720 K=1,NPTS                                                     BC003700
720 X1(K)=ALOG(ALOG(X1(K)))                                             BC003710
    GO TO 825                                                           BC003720
725 DO 730 K=1,NPTS                                                     BC003730
730 X1(K)=ALOG(ATRAN1+X1(K))                                            BC003740
    GO TO 825                                                           BC003750
735 DO 740 K=1,NPTS                                                     BC003760
740 X1(K)=ALOG(BTRAN1+ALOG(CTRAN1+X1(K)))                               BC003770
    GO TO 825                                                           BC003780
745 DO 750 K=1,NPTS                                                     BC003790
750 X1(K)=SQRT(X1(K))                                                   BC003800
    GO TO 825                                                           BC003810
755 DO 760 K=1,NPTS                                                     BC003820
760 X1(K)=1.0/X1(K)                                                     BC003830
    GO TO 825                                                           BC003840
765 DO 770 K=1,NPTS                                                     BC003850
770 X1(K)=1.0/(DTRAN1+X1(K))                                            BC003860
    GO TO 825                                                           BC003870
775 DO 780 K=1,NPTS                                                     BC003880
780 X1(K)= ATAN(X1(K)/SQRT(1.0-X1(K)**2))                               BC003890
    GO TO 825                                                           BC003900
785 DO 790 K=1,NPTS                                                     BC003910
790 X1(K)=2.0*ATAN(SQRT(X1(K))/SQRT(1.0-X1(K)))                         BC003920
    GO TO 825                                                           BC003930
795 DO 800 K=1,NPTS                                                     BC003940
800 X1(K)=X1(K)/ETRAN1                                                  BC003950
    GO TO 825                                                           BC003960
805 DO 810 K=1,NPTS                                                     BC003970
810 X1(K)=SIN(X1(K))                                                    BC003980
    GO TO 825                                                           BC003990
815 DO 820 K=1,NPTS                                                     BC004000
820 X1(K)=COS(X1(K))                                                    BC004010
825 IF (ITRAN2.LE.1) GO TO 950                                          BC004020
    GO TO (950,830,840,850,860,870,880,890,900,910,920,930,940),ITRAN2BC004030
830 DO 835 K=1,NPTS                                                     BC004040
835 X2(K)=ALOG(X2(K))                                                   BC004050
    GO TO 950                                                           BC004060
840 DO 845 K=1,NPTS                                                     BC004070
845 X2(K)=ALOG(ALOG(X2(K)))                                             BC004080
    GO TO 950                                                           BC004090
850 DO 855 K=1,NPTS                                                     BC004100
855 X2(K)=ALOG(ATRAN2+X2(K))                                            BC004110
    GO TO 950                                                           BC004120
860 DO 865 K=1,NPTS                                                     BC004130
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  865 X2(K)=ALOG(BTRAN2+ALOG(CTRAN2+X2(K)))                          BC004140
      GO TO 950                                                      BC004150
  870 DO 875 K=1,NPTS                                                BC004160
  875 X2(K)=SQRT(X2(K))                                              BC004170
      GO TO 950                                                      BC004180
  880 DO 885 K=1,NPTS                                                BC004190
  885 X2(K)=1.0/X2(K)                                                BC004200
      GO TO 950                                                      BC004210
  890 DO 895 K=1,NPTS                                                BC004220
  895 X2(K)=1.0/(DTRAN2+X2(K))                                       BC004230
      GO TO 950                                                      BC004240
  900 DO 905 K=1,NPTS                                                BC004250
  905 X2(K)=ATAN(X2(K)/SQRT(1.0-X2(K)**2))                           BC004260
      GO TO 950                                                      BC004270
  910 DO 915 K=1,NPTS                                                BC004280
  915 X2(K)=2.0*ATAN(SQRT(X2(K))/SQRT(1.0-X2(K)))                    BC004290
      GO TO 950                                                      BC004300
  920 DO 925 K=1,NPTS                                                BC004310
  925 X2(K)=X2(K)/ETRAN2                                             BC004320
      GO TO 950                                                      BC004330
  930 DO 935 K=1,NPTS                                                BC004340
  935 X2(K)=SIN(X2(K))                                               BC004350
      GO TO 950                                                      BC004360
  940 DO 945 K=1,NPTS                                                BC004370
  945 X2(K)=COS(X2(K))                                               BC004380
  950 IPAGE =50                                                      BC004390
      DO 960 I=1,NPTS                                                BC004400
      IF (IPAGE.NE.0) GO TO 954                                      BC004410
      IPAGE=50                                                       BC004420
      PRINT 951                                                      BC004430
  951 FORMAT (1H1)                                                   BC004440
  954 PRINT 955,I,X1ORIG(I),X2ORIG(I),X1(I),X2(I)                    BC004450
  955 FORMAT (1H ,I4,2X,E14.8,2X,E14.8,3X,E14.8,2X,E14.8)            BC004460
  960 IPAGE=IPAGE-1                                                  BC004470
      X1MIN=X1(1)                                                    BC004480
      X2MIN=X2(1)                                                    BC004490
      SIGX12=X1(1)**2                                                BC004500
      SIGX22=X2(1)**2                                                BC004510
      R=X1(1)*X2(1)                                                  BC004520
      X1MEAN=X1(1)                                                   BC004530
      X2MEAN=X2(1)                                                   BC004540
      X1MAX=X1MIN                                                    BC004550
      X2MAX=X2MIN                                                    BC004560
      DO 1350 I=2,NPTS                                               BC004570
      IF (X1(I).GT.X1MAX) GO TO 970                                  BC004580
      IF (X1(I).LE.X1MIN) GO TO 990                                  BC004590
      GO TO 1000                                                     BC004600
  970 X1MAX=X1(I)                                                    BC004610
      GO TO 1000                                                     BC004620
  990 X1MIN=X1(I)                                                    BC004630
 1000 IF (X2(I).GT.X2MAX) GO TO 1100                                 BC004640
      IF (X2(I).LE.X2MIN) GO TO 1200                                 BC004650
      GO TO 1300                                                     BC004660
 1100 X2MAX=X2(I)                                                    BC004670
      GO TO 1300                                                     BC004680
 1200 X2MIN=X2(I)                                                    BC004690
 1300 SIGX12=SIGX12+X1(I)**2                                         BC004700
      SIGX22=SIGX22+X2(I)**2                                         BC004710
      R=R+X1(I)*X2(I)                                                BC004720
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      X1MEAN=X1MEAN+X1(I)                                                    BC004730
      X2MEAN=X2MEAN+X2(I)                                                    BC004740
 1350 CONTINUE                                                               BC004750
      PRINT 1250, X1MIN,X1MAX,X2MIN,X2MAX                                    BC004760
 1250 FORMAT (1H1,8HMIN X1 =,E14.8,10X,8HMAX X1 =,E14.8/1H ,8HMIN X2 =,      BC004770
     1        E14.8,10X,8HMAX X2 =,E14.8)                                    BC004780
      XNPTS=NPTS                                                             BC004790
      SIGX12=SIGX12-X1MEAN**2/XNPTS                                          BC004800
      SIGX22=SIGX22-X2MEAN**2/XNPTS                                          BC004810
      R=(R*XNPTS-X1MEAN*X2MEAN)/(XNPTS*SQRT(SIGX12*SIGX22))                  BC004820
      SIGX12=  SIGX12/(XNPTS-1.)                                             BC004830
      SIGX22=  SIGX22/(XNPTS-1.)                                             BC004840
      X1MEAN = X1MEAN/XNPTS                                                  BC004850
      X2MEAN = X2MEAN/XNPTS                                                  BC004860
      SIGX1 = SQRT(SIGX12)                                                   BC004870
      SIGX2 = SQRT(SIGX22)                                                   BC004880
      PRINT 1400,R,X1MEAN,X2MEAN,SIGX12,SIGX22,SIGX1,SIGX2                   BC004890
 1400 FORMAT (1H0,2HR=,E14.8,3X,8HMEAN X1=,E14.8,4X,8HMEAN X2=,E14.8,4X,BC004900
     1        12HVARIANCE X1=,E14.8,4X,12HVARIANCE X2=,E14.8,/  1H ,     BC004910
     2        12HSTAN DEV X1=,E14.8,5X,12HSTAN DEV X2=,E14.8)               BC004920
      A1=R*SIGX2/SIGX1                                                       BC004930
      A2=X2MEAN-A1*X1MEAN                                                    BC004940
      A3=R*SIGX1/SIGX2                                                       BC004950
      A4=X1MEAN-A3*X2MEAN                                                    BC004960
      PRINT 1450,A2,A1,A4,A3                                                 BC004970
 1450 FORMAT (1H0,3HX2=,E14.8,3H + ,E14.8,4H *X1/4H X1=,E14.8,3H + ,E14.BC004980
     18,4H *X2)                                                             BC004990
      PRINT 1475                                                             BC005000
 1475 FORMAT (1H1)                                                           BC005010
      NT=0                                                                   BC005020
      DO 1600 KN=1,NN                                                        BC005030
      NT=NT+1                                                                BC005040
      IF (PDELT(KN).EQ.0.) GO TO 1550                                        BC005050
      NT1= (PE(KN)-PS(KN))/PDELT(KN)+0.0001                                  BC005060
      P(NT)= PS(KN)                                                          BC005070
      NTP1 =NT+1                                                             BC005080
      NT = NT+NT1                                                            BC005090
      PROB = PS(KN)                                                          BC005100
      DO 1500 KM=NTP1,NT                                                     BC005110
      PROB = PROB+PDELT(KN)                                                  BC005120
 1500 P(KM)= PROB                                                            BC005130
      GO TO 1600                                                             BC005140
 1550 P(NT)=PS(KN)                                                           BC005150
 1600 CONTINUE                                                               BC005160
      NPROB =NT                                                              BC005170
      DO 1700 LM=1,NT                                                        BC005180
      IS = P(LM)*1000.+.0005                                                 BC005190
 1700 C2(LM+1) = TABLEC(IS)                                                  BC005200
      NT=NT+1                                                                BC005210
      RCOMP=1./(1.-R**2)                                                     BC005220
      DO 1800 I=1,NPTS                                                       BC005230
      AA=((X1(I)-X1MEAN)/SIGX1)                                              BC005240
      AB=((X2(I)-X2MEAN)/SIGX2)                                              BC005250
 1800 Q(I)=RCOMP*(AA**2-2.*R*AA*AB+AB**2)                                    BC005260
      C2(1)=0                                                                BC005270
      IF (NPRINT.LE.0) GO TO 2650                                            BC005280
      ILINE=50                                                               BC005290
      DO 2200 J=2,NT                                                         BC005300
      JM1=J-1                                                                BC005310
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      IF (ILINE.NE.0) GO TO 1875                                        BC005320
      ILINE=50                                                          BC005330
      PRINT 1850                                                        BC005340
 1850 FORMAT (1H1)                                                      BC005350
 1875 PRINT 1900,JM1,C2(J),JM1,P(JM1)                                   BC005360
 1900 FORMAT (1H ,4H C2(,I3,4H) = ,E14.8,5X,2HP(,I3,4H) = ,F8.3)        BC005370
      ILINE=ILINE-1                                                     BC005380
 2000 DO 2200 I=1,NPTS                                                  BC005390
      IF (ILINE.NE.0) GO TO 2050                                        BC005400
      ILINE=50                                                          BC005410
      PRINT 1850                                                        BC005420
 2050 IF ((C2(J-1).GT.Q(I)).OR . (C2(J).LT.Q(I)))  GO TO 2200           BC005430
      PRINT 2100,I,X1(I),X2(I),Q(I)                                     BC005440
 2100 FORMAT (1H ,I5,2H-(,E14.8,1H,,E14.8,2H)-,E14.8)                   BC005450
      ILINE=ILINE-1                                                     BC005460
 2200 CONTINUE                                                          BC005470
      IF (ILINE.NE.0) GO TO 2250                                        BC005480
      ILINE=50                                                          BC005490
      PRINT 1850                                                        BC005500
 2250 PRINT 2300,NT,NT                                                  BC005510
 2300 FORMAT (1H0,4H C2(,I3,12H) = INFINITY,11X,2HP(,I3,12H) =     1.000)BC005520
      ILINE=ILINE-1                                                     BC005530
      DO 2500 I=1,NPTS                                                  BC005540
      IF (ILINE.NE.0) GO TO 2400                                        BC005550
      ILINE=50                                                          BC005560
      PRINT 1850                                                        BC005570
 2400 IF (Q(I).LE.C2(NT)) GO TO 2500                                    BC005580
      PRINT 2100,I,X1(I),X2(I),Q(I)                                     BC005590
      ILINE=ILINE-1                                                     BC005600
 2500 CONTINUE                                                          BC005610
 2650 PRINT 2700,IRUN,NPTS,NT                                           BC005620
 2700 FORMAT (1H1,39X,15HCHI-SQUARE TEST/1H0,18X,11HRUN NUMBER ,I3,     BC005630
     1         6H WITH ,I5,18H OBSERVATIONS AND ,I4,10H INTERVALS/1H0,  BC005640
     2         8HINTERVAL,3X,11HUPPER BOUND,6X,11HPROBABILITY,3X,      BC005650
     3         8 HOBS FREQ,7X,9HTHEO FREQ,6X,13HCHI-SQU CONTR /1H      BC005660
     4         8 HINDEX(J),6X,5HC2(J),13X,4HP(J)/)                     BC005670
      P(NT)=1.000                                                       BC005680
      CALL MOVE (0,IOBS(1),999)                                         BC005690
      DO 2850 IP=1,NPTS                                                 BC005700
      DO 2800 IC=2,NT                                                   BC005710
      IF (C2(IC).GT.Q(IP).AND.C2(IC-1).LE.Q(IP))GO TO 2750              BC005720
      GO TO 2800                                                        BC005730
 2750 IOBS(IC-1)=IOBS(IC-1)+1                                           BC005740
      GO TO 2850                                                        BC005750
 2800 CONTINUE                                                          BC005760
      IOBS(NT) = IOBS(NT)+1                                             BC005770
 2850 CONTINUE                                                          BC005780
      E(1)=XNPTS*P(1)                                                   BC005790
      DO 2900 IE=2,NT                                                   BC005800
 2900 E(IE) = XNPTS* (P(IE)-P(IE-1))                                    BC005810
 2975 CSUM=0                                                            BC005820
      DO 3000 IJ=1,NT                                                   BC005830
      IC =IJ+1                                                          BC005840
      CSC =(OBS(IJ)-E(IJ))**2/E(IJ)                                     BC005850
      CSUM = CSUM +CSC                                                  BC005860
 3000 PRINT 3100, IJ,C2(IC),P(IJ),IOBS(IJ),E(IJ),CSC                    BC005870
 3100 FORMAT (1H ,2X,I3,5X,E14.8,5X,F8.3,7X,I4,10X,F8.4,9X,F8.4)        BC005880
      K1=NT-6                                                           BC005890
      IF(K1.LE.0) GO TO 3710                                            BC005900
```

```
      PRINT 3700,CSUM,K1                                                    BC005910
 3700 FORMAT (1H0,12HCHI-SQUARE =,F12.6,6H WITH ,I3,19H DEGREES OF FREEDBC005920
     1OM/1H2)                                                             BC005930
      IF(K1.GT.0) GO TO 3730                                               BC005940
 3710 PRINT 3725                                                           BC005950
 3725 FORMAT (1H0,76HCHI-SQUARE COULD NOT BE COMPUTED BECAUSE OF INSUFFIBC005960
     1CIENT NUMBER OF INTERVALS)                                          BC005970
 3730 IF (JPLOT.LE.0) GO TO  9000                                          BC005980
      IF (JPLOT.EQ.2) GO TO 3826                                           BC005990
      NP=0                                                                 BC006000
      DO 3800 KN=1,NL                                                      BC006010
      NP=NP+1                                                              BC006020
      IF (PPDELT(KN).EQ.0.) GO TO 3775                                     BC006030
      NP1= (PPE(KN)-PPS(KN))/PPDELT(KN)+0.0001                             BC006040
      PP(NP)=PPS(KN)                                                       BC006050
      NPP1 =NP+1                                                           BC006060
      NP = NP+NP1                                                          BC006070
      PROB1 = PPS(KN)                                                      BC006080
      DO 3750 KM=NPP1,NP                                                   BC006090
      PROB1 = PROB1+PPDELT(KN)                                             BC006100
 3750 PP(KM) =PROB1                                                        BC006110
      GO TO 3800                                                           BC006120
 3775 PP(NP)=PPS(KN)                                                       BC006130
 3800 CONTINUE                                                             BC006140
      NPROB1 = NP                                                          BC006150
      DO 3825 LP=1,NP                                                      BC006160
      ISP=PP(LP)*1000.+.0005                                               BC006170
 3825 C2P(LP)=TABLEC(ISP)                                                  BC006180
      GO TO 3827                                                           BC006190
 3826 IF (NTYPES.EQ.1) GO TO 3829                                          BC006200
      PRINT 3828,JOB,IRUN,NPTS                                             BC006210
 3828 FORMAT(2H$2,10A8/2H$ ,11HRUN NUMBER ,I3,6H WITH ,I4,13H OBSERVATIOBC006220
     1NS)                                                                 BC006230
      GO TO 4100                                                           BC006240
 3827 IF (NTYPES.LE.0) GO TO 3840                                          BC006250
 3829 PRINT 3830,XID1,XID2                                                 BC006260
 3830 FORMAT (2H$2,10A8/2H$ ,10A8)                                         BC006270
      GO TO 4100                                                           BC006280
 3840 PRINT 3850,JOB,IRUN,NPTS,NP,(PPS(JJ),PPDELT(JJ),PPE(JJ),JJ=1,NL     BC006290
     1)                                                                   BC006300
 3850 FORMAT (2H$2,10A8/2H$ ,11HRUN NUMBER ,I3,6H WITH ,I4,18H OBSERVATIBC006310
     1ONS AND ,I3,9H CONTOURS/2H$ ,5(F5.3,1H(,F5.3,1H),F5.3))             BC006320
 4100 IF (NOTIC.LE.0) GO TO 4200                                           BC006330
      PNOTIC = NOTIC                                                       BC006340
      GO TO 4300                                                           BC006350
 4200 PNOTIC = 15.                                                         BC006360
 4300 X1RANG =X1MAX-X1MIN                                                  BC006370
      X2RANG =X2MAX-X2MIN                                                  BC006380
      R1=AMAX1(X1RANG,X2RANG)                                              BC006390
      D= R1/PNOTIC                                                         BC006400
      IF (D.LT.1.) GO TO 4900                                              BC006410
      DO 4500  IP=1,9                                                      BC006420
      IF (D.EQ.TABP(IP))GO TO 4600                                         BC006430
      IF (TABP(IP).LT.D.AND.TABP(IP+1).GT.D) GO TO 4700                    BC006440
 4500 CONTINUE                                                             BC006450
      IF (D.GT.TABP(10)) D=TABP(10)                                        BC006460
      GO TO 5300                                                           BC006470
 4600 ID=TABP(IP)+.005                                                     BC006480
      GO TO 4750                                                           BC006490
```

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4700 ID1=D/TABP(IP)+.9999                                              BC006500
      D=TABP(IP)*FLOAT(ID1)                                            BC006510
     GO TO 5300                                                        BC006520
4750 D=ID                                                              BC006530
     GO TO 5300                                                        BC006540
4900 IF (D.LT.TABN(1)) GO TO 5100                                      BC006550
     DO  5000  INEG=1,9                                                BC006560
     IF (D.EQ.TABN(INEG))GO TO 5100                                    BC006570
     IF (TABN(INEG).LT.D.AND.TABN(INEG+1).GT.D)GO TO 5200              BC006580
5000 CONTINUE                                                          BC006590
5100 D=TABN(1)                                                         BC006600
     GO TO 5300                                                        BC006610
5200 ID2=D/TABN(INEG)  +.9999                                          BC006620
     D  =TABN(INEG)*FLOAT(ID2;                                         BC006630
5300 DX=D                                                              BC006640
     DY=D                                                              BC006650
     XOR=X1MIN                                                         BC006660
     YOR=X2MIN                                                         BC006670
3875 YP=C5D6*(X1MAX-X1MIN)+X2MIN                                       BC006680
     IF(YP-X2MAX) 4000,5375,3900                                       BC006690
3900 X2MAX=YP                                                          BC006700
     GO TO 5375                                                        BC006710
4000 X1MAX=C6D5* (X2MAX-X2MIN)+X1MIN                                   BC006720
5375 CALL GRF (X1MIN,X2MIN,X1MAX,X2MAX,XOR,YOR,DX,DY,IRA,IRO,0,NPTS,   BC006730
    1           X1(1),X2(1),LA,LO,2H54,-1.,0)                          BC006740
     IF (JPLOT.EQ.2) GO TO  9000                                       BC006750
     IF (JPLOT.LE.0) GO TO 9000                                        BC006760
     QRT1=(X1MAX-X1MIN)/4.                                             BC006770
     X11 =X1MIN+QRT1                                                   BC006780
     X12 =X1MAX-QRT1                                                   BC006790
     HANDY(5)=X1MEAN                                                   BC006800
     HANDY(6)=X2MEAN                                                   BC006810
     TEMP1 =(X11-X1MEAN)*SIGX2/SIGX1                                   BC006820
     TEMP2 =(X12-X1MEAN)*SIGX2/SIGX1                                   BC006830
     HANDY(2)=X2MEAN+R *TEMP1                                          BC006840
     HANDY(4)=X2MEAN+TEMP1/R                                           BC006850
     HANDY(8)=X2MEAN+R *TEMP2                                          BC006860
     HANDY(10)=X2MEAN+TEMP2/R                                          BC006870
     HANDY(1)=X11                                                      BC006880
     HANDY(3)=X11                                                      BC006890
     HANDY(7)=X12                                                      BC006900
     HANDY(9)=X12                                                      BC006910
     QRT2=(X2MAX-X2MIN)/4.                                             BC006920
     X21=X2MIN+QRT2                                                    BC006930
     X22=X2MAX-QRT2                                                    BC006940
     HANDY(12)=X21                                                     BC006950
     HANDY(14)=X21                                                     BC006960
     HANDY(16)=X22                                                     BC006970
     HANDY(18)=X22                                                     BC006980
     TEMP3=(X21-X2MEAN)*SIGX1/SIGX2                                    BC006990
     TEMP4=(X22-X2MEAN)*SIGX1/SIGX2                                    BC007000
     HANDY(11)=X1MEAN+R *TEMP3                                         BC007010
     HANDY(13)=X1MEAN+TEMP3/R                                          BC007020
     HANDY(15)=X1MEAN+R *TEMP4                                         BC007030
     HANDY(17)=X1MEAN+TEMP4/R                                          BC007040
     CALL GRF (0.,0.,0.,0.,0.,0.,0.,0.,0.,1.9,HANDY(1),HANDY(2),2,2,   BC007050
    1 2H00,-1,0)                                                       BC007060
     XOMR2=1.-R **2                                                    BC007070
     DO 5500 J=1,NP                                                    BC007080
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      V=C2P(J)                                                          BC007090
      TEMP1= SIGX1*SQRT(V)                                              BC007100
      XINTVL=1.999*TEMP1/499.                                           BC007110
      XORD=X1MEAN-TEMP1-XINTVL                                          BC007120
      DO 5400 L=1,500                                                   BC007130
      XORD=XORD+XINTVL                                                  BC007140
      XTMP=(XORD-X1MEAN)/SIGX1                                          BC007150
      DISC=SQRT(ABS((V-XTMP**2)*XOMR2))                                 BC007160
      X1(2*L)=XORD                                                      BC007170
      X1(2*L-1)=XORD                                                    BC007180
      X2(2*L-1)=X2MEAN+SIGX2*(R *XTMP+DISC)                             BC007190
 5400 X2(2*L)=X2MEAN+SIGX2*(R *XTMP-DISC)                               BC007200
      CALL GRF (0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,1.,1000.,X1(1),X2(1),1,1,2H75,BC007210
     1          -1,0)                                                   BC007220
 5500 CONTINUE                                                          BC007230
 9000 CALL INTVL(Y)                                                     BC007240
      PRINT 11000,Y                                                     BC007250
11000 FORMAT (27H RUNNING TIME IN SECONDS = ,E14.8)                     BC007260
      CALL SEND(NPTS,X1ORIG,X1)                                         BC007270
      CALL SEND (NPTS,X2ORIG,X2)                                        BC007280
10000 CONTINUE                                                          BC007290
      GO TO 50                                                          BC007300
      STOP                                                              BC007310
      END                                                               BC007320
```

VIII. REFERENCES

Bates, C. E. (1966), The Chi-Square Test of Goodness of Fit for a Bivariate Normal Distribution, NWL Technical Memorandum No. K-77/66, U. S. Naval Weapons Laboratory, Dahlgren, Virginia.

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13. ABSTRACT

A description is given of the BI-CHI (Bivariate Chi-Square Test) computer program. The program is for assessing the agreement of a continuous two-dimensional sample distribution with a parent bivariate normal distribution. The program computes the Chi-square statistic for testing the null hypothesis "The random sample is from a bivariate normal parent population." The minimum and maximum sample values of each random variable are determined, and the sample estimates of the five parameters (two means, two variances, and the correlation coefficient) of the bivariate normal distribution are computed. Also, the sample estimates of the two regression lines (x_2 on x_1 and x_1 on x_2) are computed. An optional feature allows additional output of a plot of the sample data and the contour ellipses. In addition to an example problem illustrating the input and the output format, a flow chart and program listing are included in the report.

The program is coded in FORTRAN IV for the IBM 7030 (STRETCH) computer.

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